

Resonant ultrasound spectroscopy of cubes over the full range of Poisson's ratio

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Methods are developed for study of isotropic cubes via resonant ultrasound spectroscopy. To that end, mode structure maps are determined for freely vibrating isotropic cubes via finite element method over the full range of Poisson's ratio ν (-1 to $+0.5$). The fundamental torsional mode has the lowest frequency provided ν is between about -0.31 and $+0.5$. Experimental measurements for the mode structures of materials with Poisson's ratio $+0.33$, $+0.3$, $+0.15$, -0.15 , and -0.72 are performed using resonant ultrasound spectroscopy and interpreted. Methods are developed to identify pertinent modes. The experimental results match well with the analysis with the exception of some splitting of some modes because of slight material anisotropy. The effects of slight imperfection of specimen shape on the first 10 modes are analyzed for various Poisson's ratios. © 2012 American Institute of Physics. [<http://dx.doi.org/10.1063/1.4765747>]

I. INTRODUCTION

Poisson's ratio (ν) is defined as the ratio of transverse contraction strain to longitudinal extension strain in tension. The Poisson's ratio can range from -1 to 0.5 for thermodynamically stable isotropic materials, but most common materials have positive Poisson's ratio (0.25 – 0.33) as the cross section becomes thinner when stretched. Negative Poisson's ratio is known to occur for certain directions of load upon single crystals and also occurs in designed foams, via unfolding of the cells.¹ Negative values have also been observed experimentally in polymer gels^{2,3} and ferroelastic materials^{4,5} over a narrow temperature range near volumetric phase transformations. Negative Poisson's ratios in 2D systems containing rotating rigid discs in contact have been studied by computer simulations⁶ and understood via a model⁷ which has recently been generalized.⁸

Resonant ultrasound spectroscopy (RUS)⁹ can determine elastic or viscoelastic moduli by measuring the resonance structure of specimens of compact shape, for example, cubes,¹⁰ parallelepipeds, spheres, and short cylinders.¹¹ The RUS approach has advantages compared with other experimental methods to determine material properties: it is simpler in that the specimen does not need to be glued, clamped, or aligned. Numerical inversion of the mode structure to obtain elastic constants is done in the study of single crystals. The inversion process complicates the method because several iterations of alignment may be needed if some modes do not appear with sufficient amplitude to be recognized. Numerical inversion is problematical for materials with high viscoelastic damping which broadens resonances so they overlap. Inversion does not work well in materials that are not perfectly homogeneous because the higher modes become out of tune.

In such cases, a graphical method of interpretation is helpful; isotropic material properties are readily determined from the lowest few modes. Demarest¹⁰ provided a diagram of modal frequency as a function of Poisson's ratio for isotropic cubes of Poisson's ratio between 0.05 and 0.45 . With such a plot, one can easily extract Poisson's ratio as well as modulus and damping without numerical inversion. However, a Demarest plot for an isotropic cube over a full range of isotropic Poisson's ratio has not been presented. The present study is intended to fill this gap. The rationale for cubes is that they are easy to prepare from a wide variety of materials using standard tools found in a materials laboratory. That is not the case for pyramids, spheres, or tetrahedrons. Similarly, cylinders are easily cut from materials that have been cast into tubes. A cubical specimen shape is particularly appropriate for materials that cannot be cast, and for materials that are difficult to machine into other shapes.

In this work, the mode structures for a cube is determined numerically and plotted for the full range of isotropic Poisson's ratio from -0.98 to 0.48 . The effect of slight deviations from ideal cubic specimen geometry is numerically evaluated for materials with various Poisson's ratios. Results are compared with those of Demarest and with experimental results for materials of positive and negative Poisson's ratio. Methods are provided using standard transducers for the identification of modes.

The rationale for a graphical method to extract material properties from vibration mode structure is that (i) for isotropic materials it can be quicker and more straightforward than numerical methods; (ii) numerical methods require many modes for convergence, but in materials with high viscoelastic damping, higher modes overlap, preventing convergence; (iii) specimens that are not perfectly homogeneous due to polycrystalline structure or in composites or due to other causes, may exhibit higher modes that are out of tune, preventing convergence of the algorithm. A graphical

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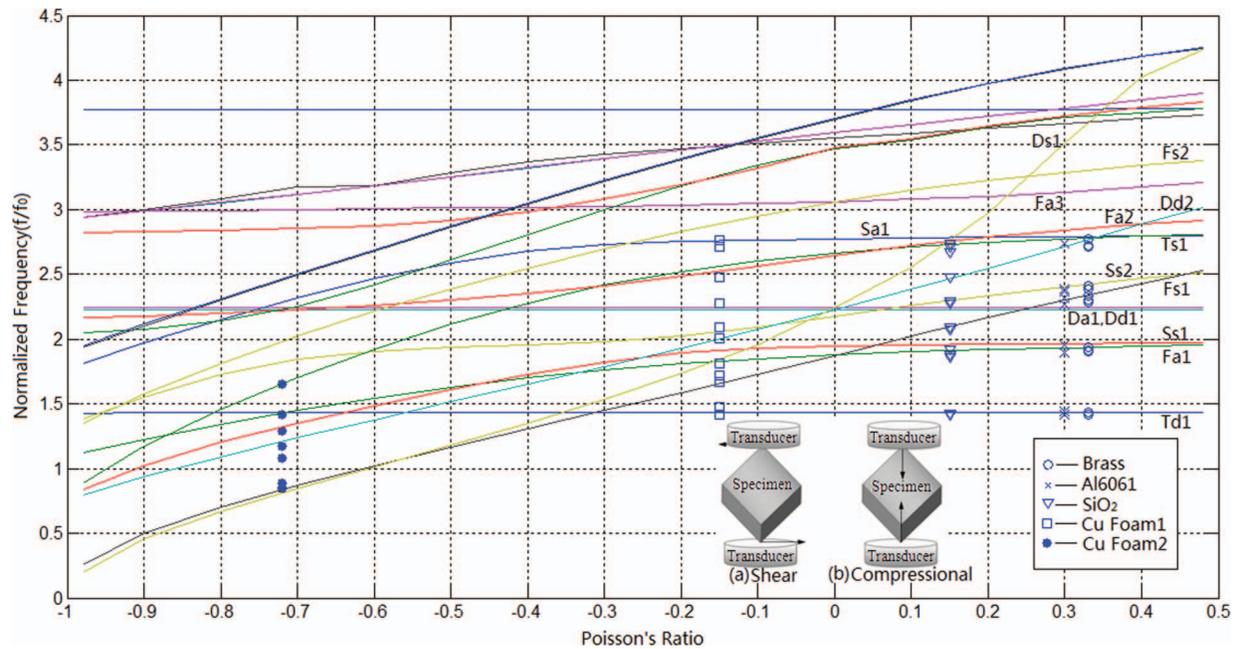


FIG. 1. Normalized frequencies of an isotropic cube over the full range of isotropic Poisson's ratio from -1 to 0.5 . The capital letters D, T, S, and F refer to: dilation, torsion, shear, and flexure, respectively; the subscripts "s," "a," and "d" refer to symmetric, anti-symmetric, and doublet, respectively; the number refers to the order of the mode. Modal frequencies are normalized to the fundamental torsional frequency. Experimental data are indicated as (\cdot) , (\square) , (∇) , (\times) , and (\circ) which represent Cu foam 2 (Cu foam squeezed by a volumetric compression ratio of 3.10), Cu foam 1 (Cu foam squeezed by a volumetric compression ratio of 1.44), SiO_2 , Al6061 alloy, and Brass, respectively.

method allows isotropic material property determination from the lowest few modes; such a method is applicable to such specimens.

The present paper also provides methods for mode identification that are appropriate for cubical specimens and that differ from methods used to identify modes in cylinders. These methods should be helpful to researchers who use either graphical or numerical methods to interpret resonant ultrasound results.

II. NUMERICAL ANALYSIS

Cubical models were created using the commercial finite element software ANSYS on a personal computer. For the mode structure as a function of isotropic Poisson's ratio, isotropic solid cubes were created using 3D, deformable solid models. Poisson's ratio was in the range from -0.98 to 0.48 . We did not cover the Poisson's ratio values of -1 and 0.5 , as these limiting values correspond to zero bulk and zero shear modulus, respectively; they are not accessible to the software package. Increments of 0.1 were used. Swept meshes of 2535 hexahedral elements (Solid 185, 8 nodes) were used. Usually, a mesh of such high density is chosen that its further increase either does not influence the results or results extrapolated to infinitely dense mesh do not differ from the obtained results by more than a chosen error. Mode shapes and frequencies were determined using modal type of analysis; Block Lanczos was selected as the mode extraction method for the first 100 modes.

III. EXPERIMENTAL

Samples with cubical shape of the following materials are prepared: Brass with dimensions $10 \times 10 \times 10.02 \text{ mm}^3$,

Al6061 alloy $10 \times 10.1 \times 9.9 \text{ mm}^3$, fused silica (amorphous SiO_2 ; Technical Glass, Painesville, OH) $6.66 \times 6.66 \times 6.64 \text{ mm}^3$, and open cell copper foams (Astro Met Associates, Inc., Cincinnati, OH) of various size. The samples were cut with a diamond saw, and sanded and polished into final dimensions. The as-received copper foam was processed by a sequence of plastic deformation in orthogonal directions to achieve appropriate permanent volumetric compression. This gives rise to negative values in Poisson's ratio due to the microbuckling of the cell ribs.¹²

Figure 1 shows the RUS sample orientation relative to the shear and compressional transducer sensitivities. The specimen was supported at its corners with minimal force to approximate the free vibration condition assumed in the analysis. Transducers used were Panametrics V153 1.0/0.5 broadband shear, polarized with center frequency 1 MHz as well as longitudinal (compressional) 1 MHz transducers. The driver transducer was excited via a synthesized function generator. Shear transducers provide a stronger signal than compressional transducers for some modes, especially for the crucial fundamental torsional mode.¹³ The output of the receiver transducer was amplified by a preamplifier. The bandpass was 100 Hz–300 kHz and the gain was from 100 to 1000. The function generator (Stanford Research DS 345) has a quoted frequency resolution of $1 \mu\text{Hz}$, and an accuracy of 5 ppm. The signals were captured by a digital oscilloscope (Tektronix TDS 3012B). Contact force was adjusted by moving one transducer with a fine micrometer drive (vertical stage, Newport type MVN50). Contact force can perturb modulus and damping measurement. So, contact force was minimized in these experiments, translating to a small systematic error.

Identification of modes was done as follows. Polarization effects on the torsion mode are not as effective for the cube

as it is for the cylinder. As the cube is rotated about its contact points, the received signal varies with a period of 120° , corresponding to the symmetry of the cube. Higher modes exhibit a similar variation but with a shift in phase. Therefore, the torsion mode was identified by placing the specimen at the center of longitudinal transducers. Assuming perpendicular alignment, the torsion mode amplitude is zero by symmetry. Experimentally the torsion mode was too weak to resolve in this case. A repeat scan is done with the contact points near the periphery of the longitudinal transducers. The torsion mode then appears. The torsion mode is also strong if shear transducers are used. The torsional mode was identified by comparing the resonant responses obtained from the shear transducers with those obtained from the compressional transducers.

IV. RESULTS AND DISCUSSION

A. Numerical results: Effect of Poisson's ratio on modes

Figure 1 shows the Demarest plot for an isotropic cube (bodies of cubic shapes made of isotropic materials) over the full range of isotropic Poisson's ratio from -1 to 0.5 . Frequencies are normalized to the following frequency:

$$f_0 = \frac{1}{\pi L} \sqrt{\frac{G}{\rho}}, \quad (1)$$

in which L is the cube side length, and ρ is the density. This is the normalization used by Demarest; the torsion mode appears in the graph at a normalized frequency $ff_0 = \sqrt{2}$. The lowest Mindlin-Lamé shear mode¹⁴ in the isotropic cube is a factor 1.57 up from the fundamental torsion mode and, as with torsion, is independent of Poisson's ratio. The Mindlin-Lamé mode frequencies are known analytically for several symmetry classes. For example, in the cubic system, with m as an integer

$$f = \frac{m}{L\sqrt{2}} \sqrt{\frac{(C_{11} - C_{12})/2}{\rho}}. \quad (2)$$

For the isotropic case, $(C_{11} - C_{12})/2 = C_{66}$ so this frequency is governed by the shear modulus alone. That is useful in the identification of modes particularly in ranges of Poisson's ratio for which the Mindlin-Lamé modes (e.g., Dd1) are well separated from other modes.

The modes shown in Fig. 1 represent the first 20 modes. Discrete values of Poisson's ratio chosen give rise to kinks in the curves. The results agree with those of Demarest for the range of Poisson's ratio for which results are given.

It can be seen from Fig. 1 that the fundamental mode for an isotropic cube is the torsional mode (i.e., Td1) when the Poisson's ratio is between approximately -0.31 and $+0.48$. For $-0.57 < \nu < -0.31$, the fundamental mode is a predominantly bending mode (i.e., Fs1). For $-0.98 < \nu < -0.57$, the fundamental mode is the first dilation mode (i.e., Ds1). The Fs1 mode has a sufficient slope with respect to Poisson's ra-

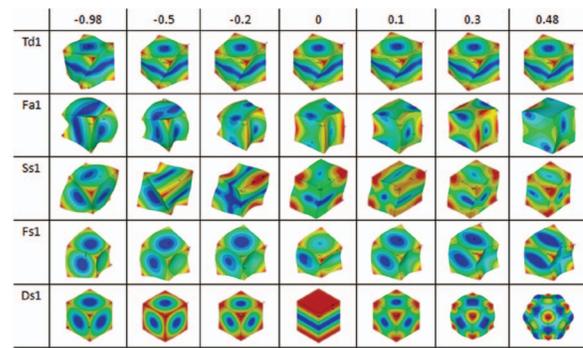


FIG. 2. Representative mode shapes and their dependence on Poisson's ratio for the first five modes including Td1, Fa1, Ss1, Fs1, Ds1. Colors represent magnitude of displacement; for interpretation of color (or shading), refer to the text.

tio to allow its determination; it is the second mode for $-0.24 < \nu < 0$, the fourth mode for $0 < \nu < 0.25$, and the sixth mode for $-0.25 < \nu < 0.46$. The lowest frequencies are particularly important for interpretation because torsion provides G alone, and the lowest modes are spaced more widely than higher ones. Modes Fs1, Dd2, Ds1 are very sensitive to Poisson's ratio; mode Fs1 is the lowest mode with substantial slope so it is the most useful to obtain Poisson's ratio.

Representative mode shapes of isotropic cubes and their dependence on Poisson's ratio for the first five modes (i.e., Td1, Fa1, Ss1, Fs1, and Ds1) are shown in Fig. 2. Colors represent the magnitude of displacement. Minimum and zero displacement appear as dark blue. Maximum displacement appears as red, and the intermediate displacements appear as yellow and green. For torsion, the maximum displacements occur at the cube corners; zero displacements occur along the axis of rotation and on the surface at the midpoint between edges.

Generally speaking, slight deviation in shape from ideal cube inevitably exists from an experimental perspective. Such deviations may include the difference in side length and deviation from right angle corners, corresponding to a monoclinic shape. It will be helpful for the interpretation of experimental results by exploring the effects of such shape deviations from the ideal cube on the mode structures.

Figure 3 shows the normalized frequencies for the first 10 modes of an isotropic cuboid with slight tetragonality as a function of aspect ratio H/L (i.e., tetragonality) for materials with various Poisson's ratios. The cuboid has side length of L and height of H (as shown in Fig. 5(a)). Most modes tend to shift to lower positions on the vertical axis when the aspect ratio H/L deviates from 1, but the decreasing rates vary from one mode to another. Other modes, such as Fa2 and Ss2, will keep a monotonic rate in lowering of position as aspect ratio H/L increases. As a result, the order of mode structures may change due to the tetragonality of the cube specimen.

Figure 4 shows the first ten modes of the cuboid with a slightly monoclinic shape (with equal sides) versus the shear deforming angle (linear art figure is shown in Fig. 5(b) for the illustration of shear deformation and angle α) at various Poisson's ratios. It can be observed from Fig. 5 that slight shear deformation has minimal effect on the normalized frequencies

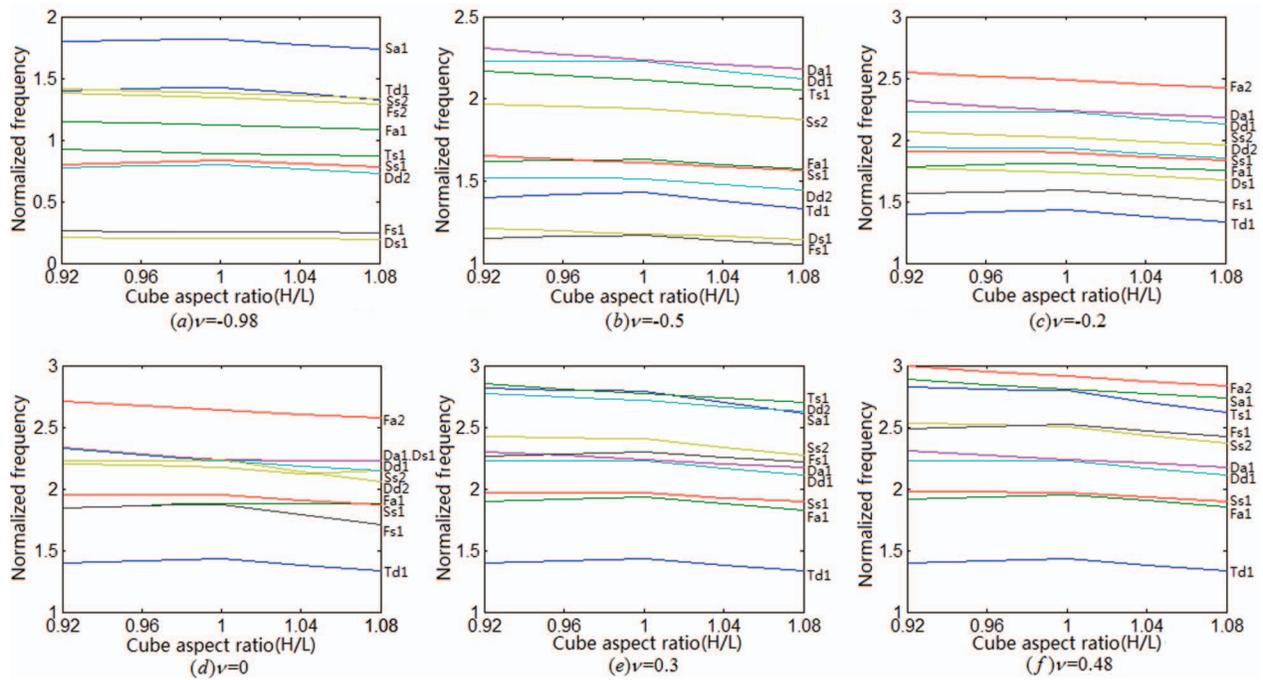


FIG. 3. Normalized frequencies for an isotropic cuboid as a function of aspect ratio H/L for materials with various Poisson's ratios (a) $\nu = -0.98$; (b) $\nu = -0.5$; (c) $\nu = -0.2$; (d) $\nu = 0$; (e) $\nu = 0.3$; and (f) $\nu = 0.48$.

for almost all the modes studied. No kink was observed on any mode as the shear deforming monoclinic angle changes (within the range from 90° to 93°). Modes monotonically shift to lower positions on the vertical axis with increasing monoclinic deformation but with different slopes. $Fs1$ and $Fa1$ have steeper slopes compared with other modes with respect to the deformation. Such an effect is considered to be attributed to the following reason. $Fs1$ and $Fa1$ correspond to the bending modes; the cuboid shear may allow bending deformation to

occur more easily, and hence makes the modes to show up at relatively lower frequencies.

B. Experimental results

The mode structures for brass, Al6061 alloy, SiO_2 , and Cu foam cube samples were determined through RUS measurements and were plotted in Fig. 1 to compare with the numerical results. Frequencies were normalized to the first

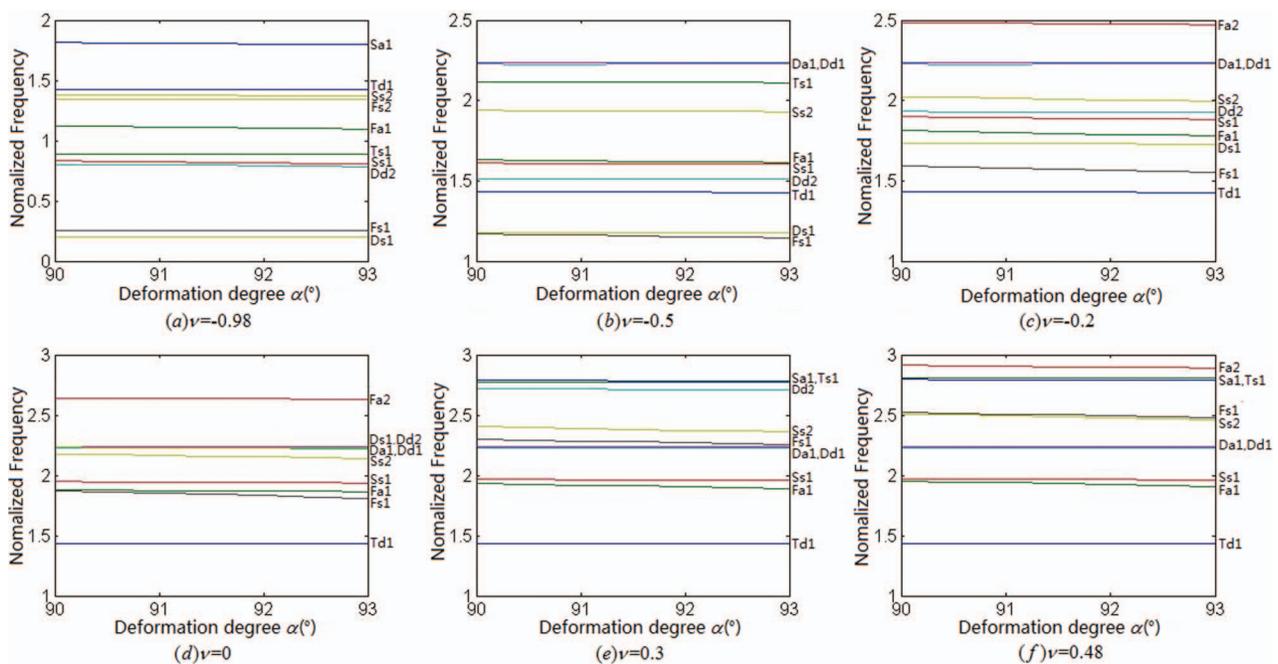


FIG. 4. (a)–(f) Normalized frequencies for an isotropic cuboid as a function of shear deforming (monoclinic) angle α for materials with various Poisson's ratios.

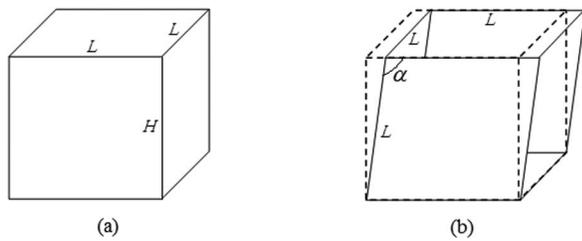


FIG. 5. A cuboid with (a) tetragonal and (b) monoclinic deformation.

torsional mode frequency. The first torsional mode is determined by the following method. The torsional mode is usually too weak to be detected by using compressional transducers, particularly if the specimen is positioned at the transducer centers. By contrast, a very sharp response can be observed by using shear transducers. For brass, Al6061 alloy and SiO₂, the first torsional mode is found to be the lowest mode that can be detected (i.e., no additional mode below the first torsional mode was found), and their torsional fundamental modes are 105.5 kHz, 145.1 kHz, and 262.8 kHz, respectively. The corresponding shear moduli derived from Eq. (1) are 36.8 GPa, 28 GPa, and 33.2 GPa, respectively, which are consistent with published results.¹¹ Compared with the Demarest plot derived from the present numerical analysis, the Poisson's ratios of brass, Al6061 alloy and SiO₂ were determined to be +0.33, +0.3, and +0.15, respectively. These results are consistent with accepted values. Also the Mindlin Dd1 mode was too weak to resolve when the specimen was at the center of longitudinal transducers, but became visible when the specimen was placed off center or tested with shear transducers.

For the two re-entrant copper foam specimens with different volumetric compression ratios of 1.44 and 3.1, the first torsional frequency and calculated shear modulus are 4.612 kHz and 75.5 MPa, 5.88 kHz and 158.7 MPa, respectively. However, the first torsional mode is not the fundamental (lowest) mode for the copper foam specimen with a volumetric compression ratio of 3.1. A non-torsional mode was observed below the first torsional mode, which is identified by the fact that it shows up by using both compressional transducers and shear transducers; the torsion mode is not detectable when compressional transducers are used and the specimen is placed at the center. The Poisson's ratios of the two re-entrant copper foam specimens were determined to be about

−0.15 and −0.72, respectively (as shown in Fig. 1). Splitting of the first mode was observed which is attributed to a slight material anisotropy. The Poisson's ratio of transformed copper foam is consistent with values reported earlier.

The plot of mode structures for a cube sample as a function of Poisson's ratio ν was generated numerically over the full range of isotropic Poisson's ratio ν (−1 to +0.5). The fundamental frequency is the torsional mode when ν is between −0.31 and +0.5, and is the bending mode when $−0.57 < \nu < −0.31$. The dilation mode becomes the fundamental frequency when $\nu < −0.57$. The Fs1 mode has a sufficient slope with respect to Poisson's ratio to allow its determination. It is the second mode when $−0.24 < \nu < 0$, the fourth mode when $0 < \nu < 0.25$, and the sixth mode when $−0.25 < \nu < 0.46$.

V. CONCLUSIONS

The present numerical results are in good agreement with the present RUS experiments for cubes with Poisson's ratio +0.33, +0.3, +0.15, −0.15, and −0.72. The results match well except for slight splitting of some modes by less than 5% because of slight material anisotropy. Modes were identified by difference in response to longitudinal vs. shear excitation, therefore, mode splitting did not interfere with interpretation. The effects of slight deviation from ideal cubical shape on the first 10 modes are analyzed for various Poisson's ratios. Results show that the effects of monoclinic deviation are small, while the effects of tetragonal deviation are of larger magnitude. Small deviations do not interfere with interpretation.

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