Design of a fastener based on negative Poisson's ratio foam
adapted from

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Abstract
In this article we make use of the negative Poisson's ratio of recently developed cellular solids in the design of a press-fit fastener. Insertion of the fastener is facilitated by the lateral contraction which negative Poisson's ratio materials exhibit under compression. Removal of the fastener is resisted by the corresponding elastic expansion under tension.

## 1	Introduction

The invention of a new class of negative Poisson's ratio foams has been recently reported[1-3]. These are in contrast to most ordinary materials, including most cellular solids, which have Poisson's ratios near 0.3. Metallic and polymeric foams were transformed so that their cellular micro- structure became re-entrant, i.e. with inwardly protruding cell ribs. Re-entrant foams exhibited negative Poisson's ratios as well as greater resilience than conventional foams. The polymer foams exhibited negative Poisson's ratios as small as -0.7, and metal foams exhibited values to -0.8. There are other structural possibilities for achieving negative Poisson's ratios, and these are reviewed in [4].

Applications of materials with negative Poisson's ratios may be envisaged (i) based on the Poisson's ratio, (ii) based on the superior toughness, resilience, and tear resistance which has been observed in these materials, and (iii) based on the acoustic properties associated with the vibration of ribs in the material. The Poisson's ratio influences deformation kinematics in ways which may be useful, and it influences the distribution of stresses. For example, stress concentration factors are reduced in some situations, increased in others, when Poisson's ratio is negative. Materials with negative Poisson's ratios can quantitatively improve the performance of applications such as wrestling mats and cores for sandwich panels; these will be described elsewhere. We consider here the design of a press-fit fastener based on a negative Poisson's ratio material, in which the Poisson's ratio itself is relevant.

2. Design of the fastener
2.1 Basic concept; consideration of material nonlinearity

The fastener consists of a hollow circular cylindrical core, made of negative Poisson's ratio material, and a socket with a corresponding cylindrical hole. The fastener is made bigger than the hole by a tolerance t in the radial dimension. As it is inserted, the longitudinal force of insertion causes its diameter (transverse dimension) to become smaller by virtue of the negative Poisson's ratio. With sufficient force, the core diameter becomes equal to the hole diameter and the core can be inserted into the hole. Axial force applied to remove the fastener causes the core to expand laterally, generating an increased frictional force which resists removal.

All negative Poisson's ratio materials studied to date exhibit a nonlinear relationship between the Poisson's ratio   and the longitudinal strain $_z$. For the purpose of analysis, the observed nonlinear dependence on strain (Fig. 1) is approximated as a quadratic in the interval -b

$_z$ b. The parameters a and b describe the form of nonlinearity and are to be obtained from experiment.

$$\epsilon = \epsilon_z{}^2 \frac{a}{b^2} - a \tag{1}$$

Poisson's ratio is defined as $\nu = -\epsilon_r/\epsilon_z$, with ϵ_r as the radial strain. So,

$$\epsilon_r = -\nu\epsilon_z = -\frac{a}{b^2}\epsilon_z{}^3 + a\epsilon_z \tag{2}$$

The cubic relationship between radial strain and longitudinal strain exhibits relative maxima in the magnitude of the radial displacement at the surface for values of ϵ_z defined as $\pm c$; with $c = \pm b/\sqrt{3}$. Even though the material has a negative Poisson's ratio for strains from $\epsilon_z = 0$ to c to b, the radial displacement decreases for $\epsilon_z > c$. The radial component of displacement u_r is given by

$$\frac{du_r}{dr}\Big|_{u_{max}} = -\frac{ac^3}{b^2} + ac \tag{3}$$

with $u_{max} = \frac{2ab}{3\sqrt{3}}(r_o - r_i)$

The maximum radial displacement must be greater than the tolerance t, consequently, in view of the assumed material nonlinearity,

$$(r_o - r_i) \ge \frac{3t\sqrt{3}}{2ab} \tag{4}$$

2.2 Forces of insertion and removal

Now consider, in an elementary vein, the forces associated with insertion and removal of the fastener core, which is assumed to have a tangent Young's modulus of E_1 at the maximum longitudinal strain occurring during insertion. The force F_i required to insert the fastener will be the force which causes sufficient longitudinal strain ϵ_z to achieve a radial contraction equal to t. So,

$$F_i = \pi(r_o{}^2 - r_i{}^2)E_1\epsilon_z \tag{5}$$

But

$$\epsilon_r = -\nu\epsilon_z = \frac{t}{(r_o - r_i)} \tag{6}$$

So the insertion force is

$$F_i = -\pi(r_o + r_i)E_1\frac{t}{\nu} \tag{7}$$

The force F_{rm} required for removal of the fastener arises from friction with the cylindrical hole. The friction force is distributed over the region of contact of length h, and an increment of this force over region dz is

$$dF_{rm} = 2\pi(r_o - t)\mu\sigma_r(z)dz \tag{8}$$

in which μ is the coefficient of friction, σ_r is the radial stress

The radial stress σ_r is obtained via an (approximate) Hookean analysis.

$$\sigma_r(z) = E_2\epsilon_r(z) = E_2(\epsilon_z(z) - t/(r_o - r_i)) \tag{9}$$

in which E_2 is the tangential modulus as Young's modulus at a certain ϵ_z

Therefore,

$$F_{rm} = 2\pi (r_O - t) \mu E_2 \int_0^h (\nu_z(z) - t/(r_O - r_i))dz \tag{10}$$

The maximum force for removal of the fastener is limited by the strain $\nu_z(z) = \nu_c = b/3$ which gives the maximum contact force. If the fastener material is sufficiently strong, the maximum removal force is:

$$F_{rm,max} = \mu E_2 2\pi (r_O - t)h \left[\frac{b}{3} - \frac{t}{r_O - r_i} \right] \tag{11}$$

Observe that since ν is negative, the first term in the [] brackets also contributes to the removal force in the same sense as the effect of preparing an oversize core, as embodied by the second term. The removal force depends on both the Poisson's ratio ν and on the measure b of nonlinearity. If b could be made larger, ie a Poisson's ratio less dependent on strain, the removal force would be larger.

2.2 Specific example: copper re-entrant foam

Copper foam is chosen since it is relatively stiff (E $\approx$ 300 MPa), and is ductile enough to be easy to process. For copper re-entrant foam with an initial relative density of 0.1 and a permanent volumetric compression ratio of 2.13 and at a strain of 0.001, ν = -0.8 $\pm$ 0.05. The Poisson's ratio is assumed to be negative up to a strain of 0.2 [3]. Therefore constants above are chosen as a = 0.8 and b = 0.2.

Consider first the case of a small radial tolerance t=0.01mm; let h=30 mm and r_O=10 mm. Then from Eq. 4, r_i $\approx$ 9.8 mm; we choose r_i = 3 mm. We obtain ν_z = 0.18% as the strain required for insertion. The unlubricated friction coefficient of copper on steel is 0.22 [5]. The tangential Young's moduli E_1, E_2 at longitudinal strains of 0.18% and 0.2% are both about 30 MPa for copper foam[3]. Therefore, the force required to insert this fastener is F_i = 15 N; and to remove it, F_r = 112 N.

Consider now the case of a larger radial tolerance t=0.05mm. We calculate r_i $\approx$ 8.8 mm, and choose r_i=5 mm. For this case ν_z = 0.025 is the insertion strain. With E_1, E_2 of 40 MPa and 10 MPa respectively and corresponding Poisson's ratios of -0.79 and -0.51 [3] at strains of 0.025 and 0.12, the insertion force is calculated as F_i = 119 N and the removal force is F_r = 211 N. In this case the material nonlinearity limits the removal force.

3   Experimental Materials and Methods

In this experiment, the performance of a fastener made of re-entrant negative Poisson's ratio copper foam was evaluated by a simple push-pull test.

A re-entrant copper foam specimen was prepared as a core. The material had an initial relative density (density of foam divided by density of solid copper) of 0.03, a permanent volumetric compression ratio of 2.5, and a final Young's modulus of 55 MPa. It was machined into a hollow cylindrical form (outside diameter 18.24 mm, inside diameter 6.35 mm, length 55 mm) and polished with abrasive grit as fine as No. 1000. The socket was machined from steel with a hole of diameter 80 μm less than the outside diameter of the cylindrical core, so that t = 40 μm; the hole length was 25.5 mm. The machining tolerance was 13 μm. The fastener was annealed at 500 °C for 1 hr to minimize effects of cold work. The end of the core not inserted in the socket was embedded 20 mm deep in polymethyl methacrylate (PMMA) to form a grip; PMMA is much more rigid (E $\approx$ 3 GPa) than the copper foam (E $\approx$ 55 MPa).

The fastener was tested using a servohydraulic testing machine with a 2.5 kN load cell (MTS Corp.) to insert and remove the fastener core while monitoring the associated forces. Alignment for insertion was achieved by placing a low friction (Teflon) tape between the socket and the machine platen. For removal of the fastener core, the socket was fixed to the platen. Experiments were performed at a constant displacement rate of 0.1 mm/sec.

4   Experimental Results

Figure 2 shows the observed dependence of load on displacement for insertion and for removal of the fastener. During insertion, the load-displacement graph showed an initial sharp increase in load and then a more gradual increase in load until the maximum displacement was reached. The load-displacement curve showed a small fluctuation of up to 2.5N in load. This suggests a stick-slip frictional force or possibly a slight amount of crushing of the surface cell ribs in the core.

The fastener broke at a displacement of 2.8 mm and a load of 160N during the pull-out process. The fracture surface was in the body of the core, away from the socket and embedded grip ends. Moreover, the fracture surface was rough. The experimental results for maximum insertion and removal force were 70N and 160N, respectively. In comparison, the theoretical predictions were 60 N and 210 N, respectively. The observed maximum removal force was limited by the strength of the copper foam, not by friction, so it is reasonable that the observed removal force was less than the theoretical maximum friction force.

5   Discussion and conclusions

Foamed material is considerably less stiff and strong than the same material in solid form. Therefore, the strength of a fastener based on negative Poisson's ratio cellular solids will be less than that of a corresponding solid screw of the same diameter. In the present study, a foam containing only 0.03 x 2.5 or 7.5% solid material was used; a denser foam material would provide greater strength. The difference in strength could be compensated for by designing a fastener of larger diameter than would otherwise be used. The advantage of a negative Poisson's ratio fastener is the simplicity of the press-fit insertion. The example presented here of copper foam was chosen since it was readily available, is stiffer (E   55 MPa) than the elastomeric polymer foams previously examined (E   70 kPa) and its ductility facilitates transformation into a re-entrant structure. Fasteners based on stiff polymer foam are possible as well. Materials with negative Poisson's ratios of considerably higher stiffness and strength than those considered here are possible based on the concept of re-entrant cells [1,3] as well as other structures[4], so the design presented here is intended as illustrative rather than optimal.

We conclude that press-fit type fasteners may be designed using negative Poisson's ratio materials, and that such fasteners perform in a manner consistent with a simple analysis.

6   References

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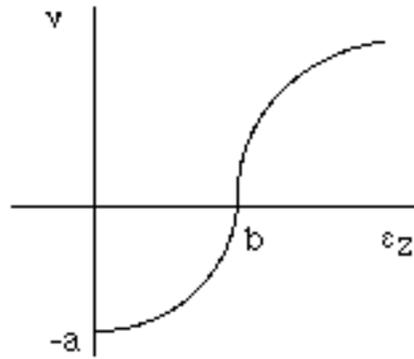
List of figures

Fig. 1   Poisson's ratio vs longitudinal strain for   re-entrant foam
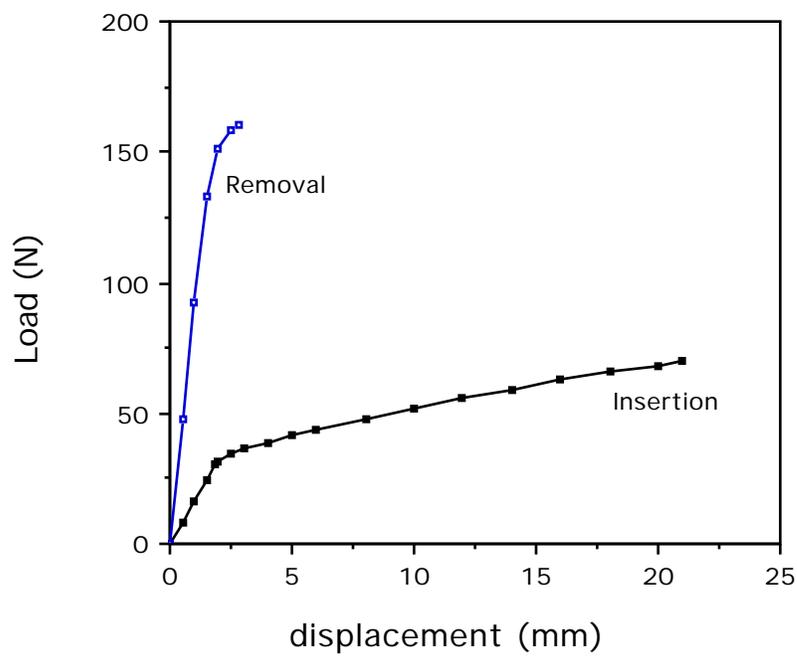

Fig. 2 Load vs displacement for experimental insertion and removal of fastener core.