Numerical analysis on mechanical behaviors of hierarchical cellular structures with negative Poisson's ratio

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Dong Li[a*], Jianhua Yin[a], Liang Dong[b], and Roderic S. Lakes[c]

[a] *College of Sciences, Northeastern University, Shenyang 110819, PR China*

[b] *Materials Science and Engineering, ,University of Virginia, Charlottesville, VA 22904, USA*

[c] *Department of Engineering Physics, University of Wisconsin, Madison, WI 53706-1687, USA*

*Corresponding author: Tel. +86 24 83678347; fax: +86 24 25962434; Email: lidong@mail.neu.edu.cn.

Abstract: A two-dimensional (2D) hierarchical re-entrant honeycomb structures were designed and the mechanical behaviors of the structures were studied using FEM (finite element method). Hierarchical structure of first order was constructed by replacing each vertex of a re-entrant hexagonal structure (zeroth order hierarchical structure) by a smaller re-entrant hexagon with identical strut aspect ratio. A second order hierarchical structure was constructed in the same manner by replacing each vertex of the first order hierarchical structure with a smaller re-entrant hexagon. The Poisson's ratio and energy absorption capacity of re-entrant structures of different hierarchical orders were studied at different area compression ratios. The results show that the aspect ratios have a strong influence on the Poisson's ratio of first and second order hierarchical structure. The Poisson's ratio of the first and second order hierarchical structures can reach -1.36 and -1.33 with appropriate aspect ratio, 13.8% and 12.1% lower than that of the zeroth order hierarchical structure. The energy absorption capacity of the three models increased with an increasing compression velocity; The second order hierarchical structure exhibited the highest rate of increase in energy absorption capacity with an increasing compression velocity. The plateau stresses of the first order and second order hierarchical structures were much lower than that of the zeroth order hierarchical structure; stress was higher for second order at high rate.

Keywords: Hierarchical structure; Re-entrant; Cellular; Negative Poisson's ratio; Energy absorption

1. Introduction

Cellular solids are widely used for many structural applications due to the low weight and high energy absorption capability. Hierarchical honeycomb structures are known to have advantages in their mechanical properties compared with regular honeycomb structures. For example, honeycomb with ribs that are themselves honeycomb exhibit much higher specific strength than conventional honeycomb [1]. Hierarchical structures designed with tubular elements also exhibit superior strength [2]. Many kinds of hierarchical honeycombs with different levels of hierarchy are proposed to optimize the physical properties of the structures [3–5]. Self-similar isotropic hierarchical honeycombs were designed by replacing each three-edge vertex of a base hexagonal network with a similar but smaller hexagon of the same orientation [6, 7]. These hierarchical honeycombs were stiffer in-plane than traditional honeycombs. The out of plane stiffness is not enhanced. Mechanisms for transverse compression and shear collapse of a first and second order hierarchical structures, whose homogeneous cell walls were replaced by trusses, were studied and found the strength of the second order hierarchical structure is about ten times greater than that of the first order hierarchical structure [8]. Functionally graded hierarchical honeycombs exhibit an increase of the elastic modulus of up to 75% compared with that of the conventional non-hierarchical counterpart [9].

Negative Poisson's ratio occurs in designed cellular materials with re-entrant structures in both 2-D [10-12] and 3-D structures [13, 14]. Negative Poisson's ratio behavior has been observed experimentally in polymer gels near the phase transitions [15, 16], in orthorhombic alloy in a set of planes [17], and in ferroelastic ceramic [18] and InSn alloy [19] near the phase transformations. A large number of works have showed that the honeycombs with negative Poisson's ratio have better mechanical behaviours than those of traditional honeycombs[20]. Incorporating hierarchy into the re-entrant honeycomb lattice structures is of interest. Auxetic behavior was demonstrated [21, 22] in several kinds of hierarchical honeycombs under uniaxial compressive loads through a combination of numerical simulations and experiments. However in these honeycombs, negative Poisson's ratio can occur only at large deformations and be obtained with special parent materials. A hierarchical tube with a negative Poisson's ratio [23] was designed as a fiber for tougher composites; the Poisson's ratio of the designed hierarchical NPR tube showed weak dependence on the geometric dimension and hierarchical order. Up to now, there have been few reports on hierarchical honeycombs with negative Poisson's ratio and none on their energy absorption capacity.

In this work, A two-dimensional (2D) hierarchical re-entrant honeycomb structures were designed. Their mechanical behaviors were studied using FEM (finite element method). Hierarchical structure of first order was constructed by replacing each vertex of a re-entrant hexagonal structure (zeroth order hierarchical structure) by a smaller re-entrant hexagon with identical strut aspect ratio. second order hierarchical structure was constructed via the same manner by replacing each vertex of the first order hierarchical structure with a smaller re-entrant hexagon. The Poisson's ratio and energy absorption capacity of re-entrant structures of different hierarchical orders were studied at different area compression ratios. The results show that the aspect ratios have strong influence on the Poisson's ratio of first and second order hierarchical structure. The Poisson's ratio of the first and second order hierarchical structures can reach -1.36 and -1.33 with appropriate aspect ratio, 13.8% and 12.1% lower than that of the zeroth order hierarchical structure. Poisson's ratio smaller than -1 is possible in anisotropic materials and structures. The energy absorption capacity of the three models increased with an increasing compression velocity; yet, the second order hierarchical structure exhibited the highest increasing rate in energy absorption capacity with an increasing compression velocity. Meanwhile, the plateau stresses of the first order and second order hierarchical structures were much lower than that of the zeroth order hierarchical structure for lower strain rates, and higher for second order hierarchy and high rates.

This work provides an insight of the role of structural hierarchy in designing 2D auxetic metamaterials, and new opportunities for developing energy absorbing materials, tunable membrane filters, and acoustic dampers.

2. Materials and methods

The geometry of the unit cell of zeroth order (n=0) re-entrant honeycomb structure, i.e., zeroth order re-entrant hexagon, is defined by its vertical length, H_0, horizontal length, L_0, and the length, L_1, which determine the degree of re-entrant (α), as shown in Fig. 1(a). In this work, it is assumed that $H_0=L_0$, and $\alpha=L_1/H_0$. The first order (n=1) hierarchical unit cell was achieved by replacing the six vertices of the zeroth order re-entrant hexagon with smaller re-entrant hexagons (zeroth order) of height H_1 and the same aspect ratio, Fig. 1(b). The second order (n=2) hierarchy was achieved in the same manner of replacing the vertices of the first order re-entrant structure with smaller hexagons of height H_2 and the same aspect ratio, Fig. 1(c). The geometric relation between structures of different hierarchy can be defined by the ratio of heights of their unit cell hexagons: $\beta_1=H_0/H_1$, $\beta_2=H_1/H_2$. Lattice structures of different hierarchies made from periodically packing the unit cells of the corresponding hierarchy are shown in Fig. 1(d-f). The overall density of the lattice structure with different order of hierarchy was kept the same by changing the truss member wall thickness. In this work, the wall thickness of the first order and second order hierarchical structures are 0.834mm, 0.577mm and 0.424mm, respectively. The rationale was to maintain an identical mass of material in the different structures, to facilitate comparisons.

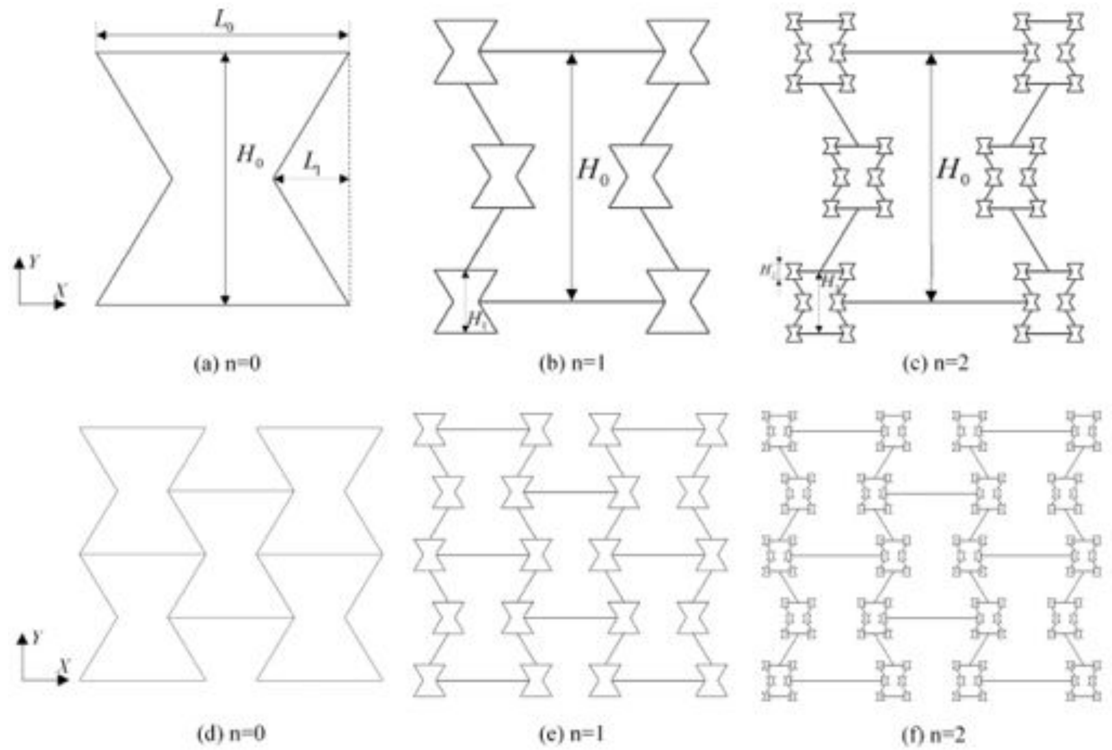

Figure 1. The unit cells of re-entrant structures of (a) zeroth order (n=0), (b) first order (n=1) and (c) second order (n=2) hierarchy. The corresponding re-entrant hierarchical lattice structures of zeroth order (n=0) first order (n=1) and second order (n=2) are shown in (d),(e) and (f)

The size of the zeroth order (n=0) re-entrant honeycomb structure model was 100x100cm with 162 cells. The properties of parent material were selected as copper. The rationale is to provide a ductile material for which there is experimental background (in foams). The Poisson's ratio is independent of these properties and depends only on geometry. The crush strength of the structure scales with the parent material strength. The density, Young's modulus, Poisson's ratio were 8930 kg m^{-3}, 117GPa, 0.35, respectively. A bilinear strain-hardening relationship was used to represent the true stress–strain relationship of the parent material and the yield strength and tangent modulus were 400MPa, 100MPa, respectively. The cell walls were meshed with Shell 163 elements(the number of elements depends on the edge length of the cell wall, every edge had at least 5 elements). The Poisson's ratio was calculated in the elastic region of the stress strain diagrams from the FE analysis according to the definition of Poisson's ratio.

ANSYS/LS-DYNA was used to analyze the energy absorption capacity of the re-entrant hierarchical structures at different compression velocities. The re-entrant hierarchical structures were compressed quasi-statically between two rigid platens; the bottom platen was fixed, and the top platen was moving at a constant velocity.

3. Results and discussion

3.1 Poisson's ratio

The relationship between the Poisson's ratio of re-entrant hierarchical structures and β_1 is shown in Fig. 2 for each order of hierarchy (β_1=β_2). When β_1=1(n=0), the Poisson's ratio is -1.187. It can be seen from Fig. 2 that the Poisson's ratios of the first and second order re-entrant hierarchical structures were first decreased to a minimum value and subsequently increased with increasing β_1. The minimum Poisson's ratios of the first and second order hierarchical structures were achieved at β_1=6 with values of approximately -1.36 and -1.33,13.8% and 12.1% lower than that of the zeroth order hierarchical structure .

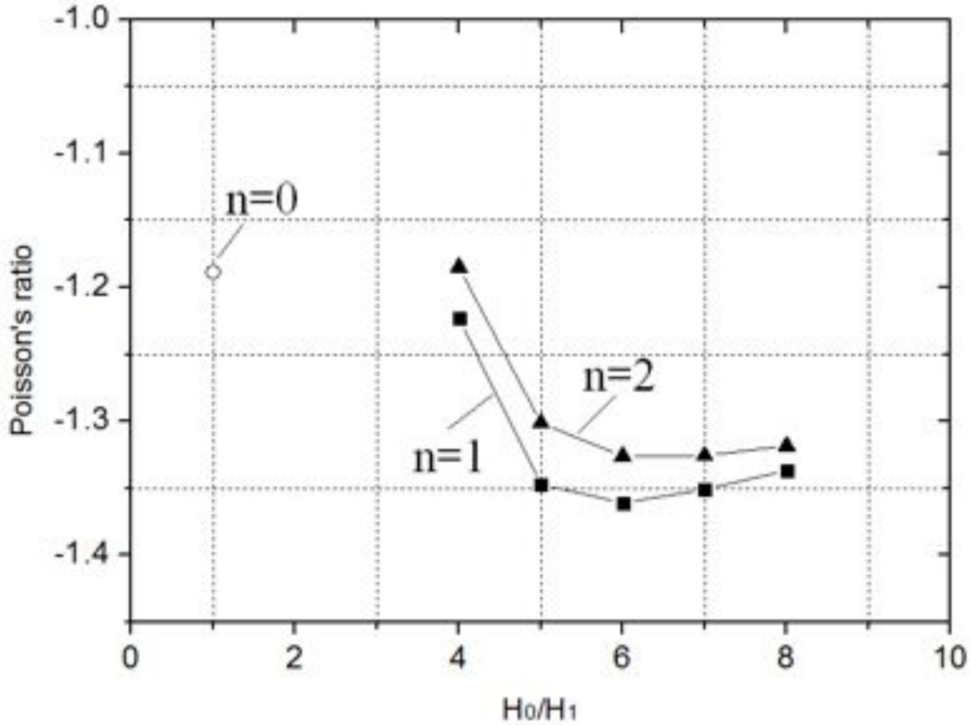

Figure 2. The relationship between the Poisson's ratio of re-entrant hierarchical structures and β_1 (β_1=H_0/H_1).

Fig. 3 shows the relationship between the Poisson's ratio of the second order re-entrant hierarchical structure and β_2/β_1. From Fig. 3 we can find that the Poisson's ratio decreased rapidly with an increasing β_2/β_1 from 0.5 to 1.0; in this β_2/β_1 range, the Poisson's ratio decreases with an increasing β_1 at each specific β_2/β_1; the decreasing rate of Poisson's ratio with β_2/β_1 decreased with an increasing β_1. The Poisson's ratios converged to a constant value of about -1.3 in all cases when β_2/β_1 >1.0.

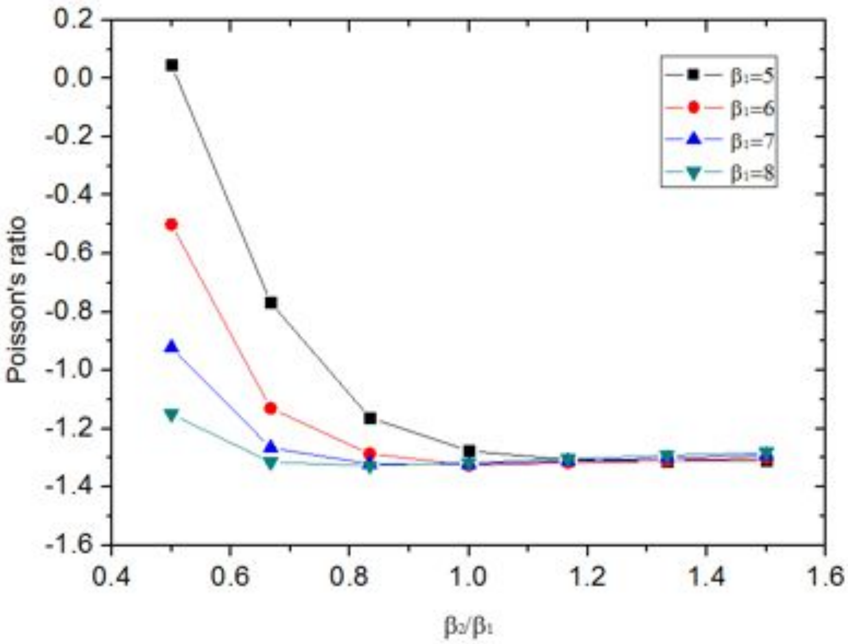

Figure 3. The relationship between the Poisson's ratio of second order re-entrant hierarchical structure and β_2/β_1 at different β_1.

3.2 Energy absorption capacity

The energy absorption capacities of the first and second order re-entrant hierarchical structures were analyzed at a constant velocity (v=10m/s) at different β_1 and β_2, as shown in Fig. 4.

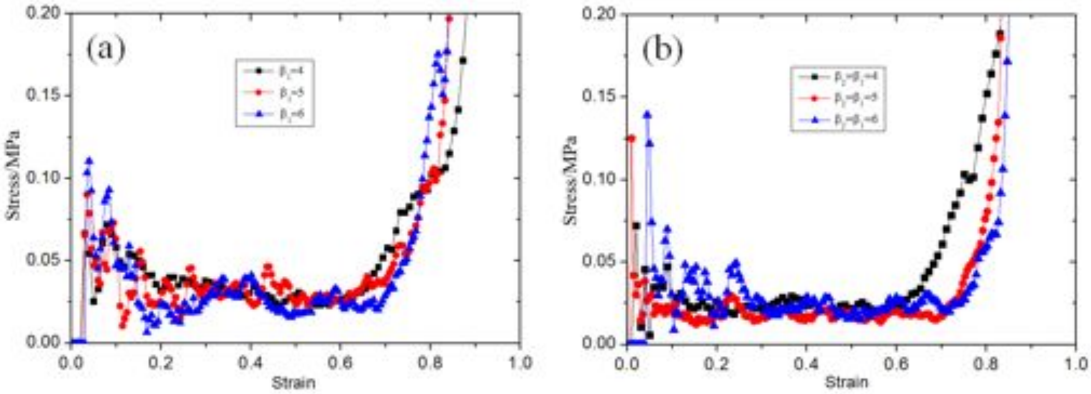

Fig. 4 The stress -strain curves of the re-entrant hierarchical structures of (a) first order (n=1) and (b) second order (n=2) at different β_1 and β_2.

The energy absorption per unit volume (W_v) of the re-entrant hierarchical structure shown in Figure 4 was calculated based on the following equation

$$W_v = \int_0^{\varepsilon_D} \sigma(\varepsilon)\mathrm{d}\varepsilon \tag{1}$$

where $\sigma(\varepsilon)$ is the flow stress of the structure; ε_D is the densification strain[23]. The results have been shown in Table 1. Results show that when $\beta_1=\beta_2=6$, the re-entrant hierarchical structures of the first and second order can exhibit the minimum negative Poisson's ratio and the highest energy absorption capacity; similar results have been reported in Ref [20].

Table 1 The Poisson's ratio ν and energy absorption per unit volume of the re-entrant hierarchical structures of the first order (n=1) and second order (n=2) at different β_1 and β_2 at v=10m/s.

	Order of hierarchy					
	n=1			n=2		
$\beta_1=\beta_2$	4	5	6	4	5	6
ν	-1.218	-1.341	-1.356	-1.185	-1.276	-1.326
W_v/MJm^{-3}	0.0253	0.0258	0.0260	0.0200	0.201	0.0255

The energy absorption capacities of the re-entrant hierarchical structures of zeroth, first and second order were analyzed at different compression velocities when $\beta_1=\beta_2=6$; their stress-strain diagrams are shown in Fig. 5.

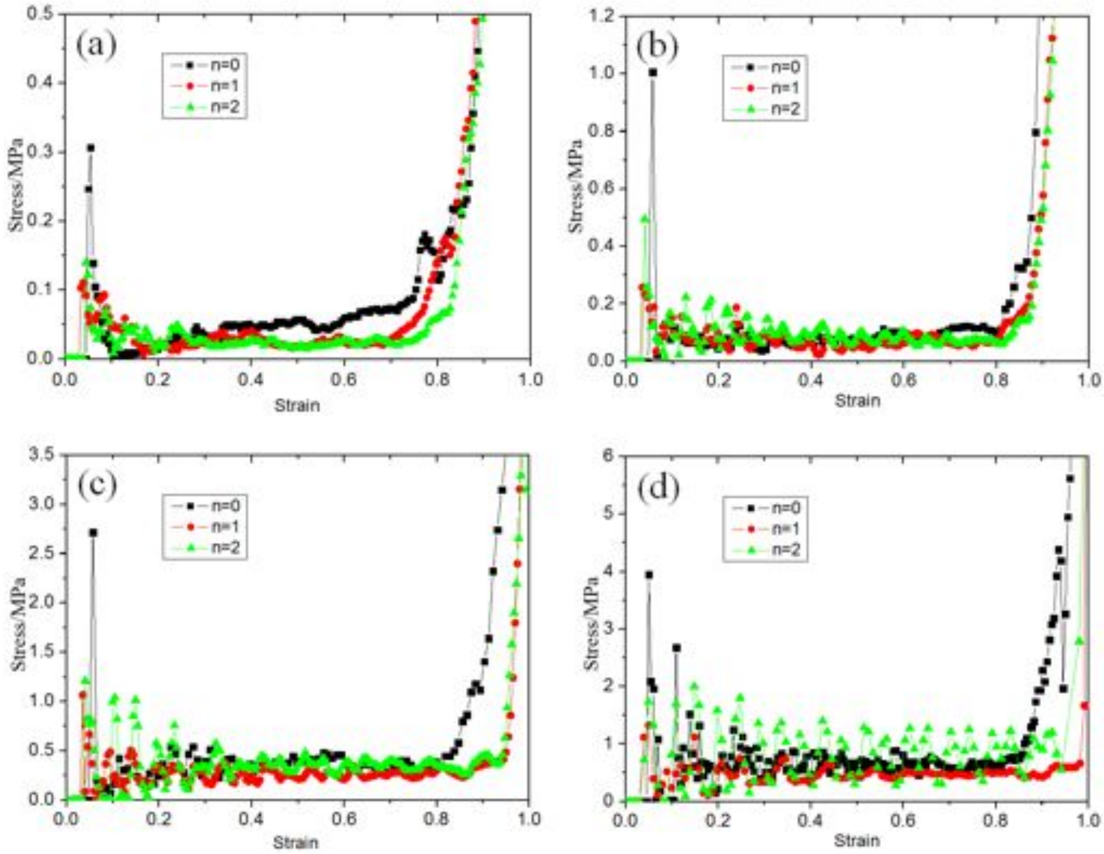

Fig. 5 The stress -strain curves of the re-entrant hierarchical structures of zeroth, first and second order(n=0,1and 2) with identical mass at (a) v=10m/s, (b) v=20m/s, (c) v=40m/s, (d) v=60m/s.

Values of the energy absorption per unit volume of the re-entrant hierarchical structures of zeroth, first and second order at different compression velocities have been shown in Table 2. Results showed that the plateau stresses of hierarchical structures of first and second order were much lower than that of the zeroth order hierarchical structure and the plateau regions of the first and second order were longer; the energy absorption capacity of the three models increased with an increasing compression velocity; similar trend has been reported by other researchers [24, 25]. At v=60m s^{-1}, the energy absorption capacity of the second order re-entrant hierarchical structure was better than that of zeroth and first order. Performance at even higher compression rate (e.g., 80m/s) has also been studied.Results showed that the energy absorption capacity of the second order re-entrant hierarchical structure was better than that of the zeroth and first order; however, the deformation mechanism was hard to observed at such a high rate, therefore, results are not shown here.

Table 2 The energy absorption per unit volume of the re-entrant hierarchical structures of zeroth, first and second order under different compression velocities.

Compression velocity(m/s)	W_v/MJm^{-3}		
	n=0	n=1	n=2
10	0.0564	0.0260	0.0255
20	0.1377	0.0715	0.0889
40	0.4140	0.2650	0.3369

60	0.6985	0.4731	0.7426

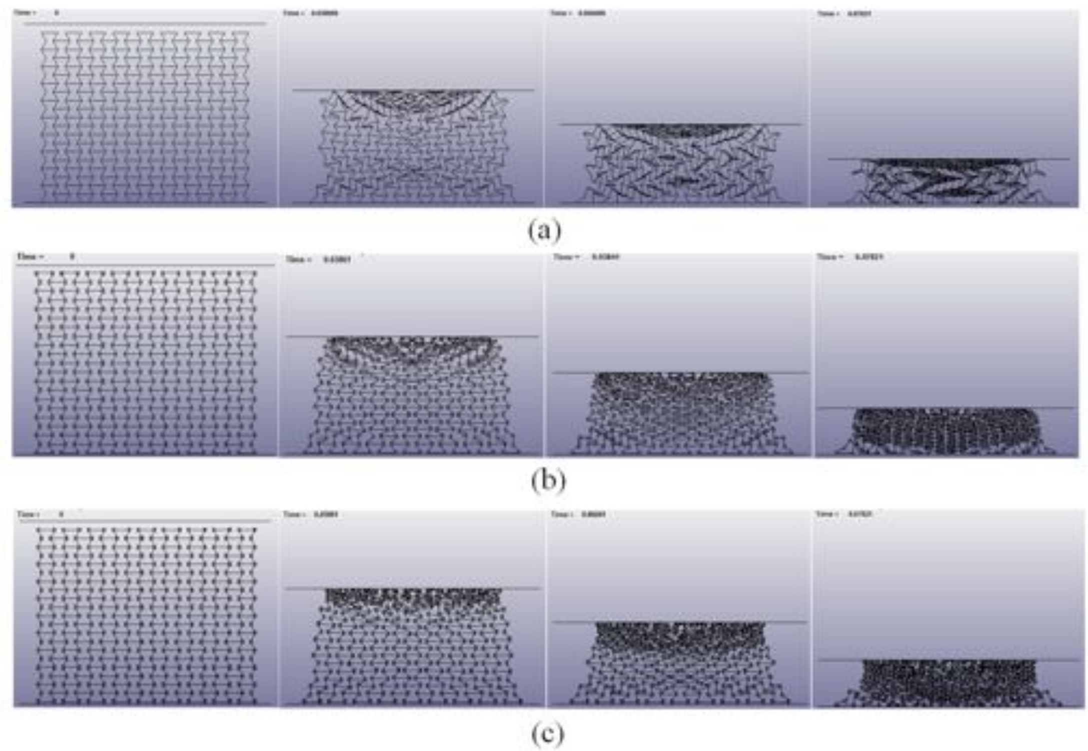

Fig. 6 Deformation mechanisms of the re-entrant hierarchical structures of (a) zeroth order, (b) first order and (c) second order at a compression velocity v=10m s^{-1}.

The deformation mechanisms of the re-entrant hierarchical structures of zeroth, first and second order at a compression velocity v=10m s^{-1} are shown in Fig.6. From Fig. 6 we can find that the "V"-shaped localized bands occur at the crushing edge of the zeroth order re-entrant hierarchical structures. For the first order re-entrant hierarchical structures, small "V"-shaped localized bands initiated from the crushing edge of the model, and propagated step by step at the crushing edge. For the second order re-entrant hierarchical structures, the collapses occurred step by step at the crushing edge, and no obvious localized deformations was observed during the compression deformation, and it is considered to be the reason of higher energy absorption capacity compared with the zeroth and first order hierarchical structures.

4. Conclusions

A two-dimensional (2D) hierarchical re-entrant honeycomb structures were designed and the mechanical behaviors of the structures were studied using FEM (finite element method). Hierarchical structure of first order was constructed by replacing each vertex of a re-entrant hexagonal structure (zeroth order hierarchical structure) by a smaller re-entrant hexagon with identical strut aspect ratio. second order hierarchical structure was constructed via the same manner by replacing each vertex of the first order hierarchical structure with a smaller re-entrant hexagon. The results show that the aspect ratios have strong influence on the Poisson's ratio of the first and second order hierarchical structure. The Poisson's ratio of the first and second order hierarchical structures can reach -1.36 and -1.33 with appropriate aspect ratio, 13.8% and 12.1% lower than that of the zeroth order hierarchical structure. The energy absorption capacity of the three models increased with an increasing compression velocity; yet, the second order hierarchical structure exhibited the highest increasing rate in energy absorption capacity with an increasing compression velocity. Meanwhile, the plateau stresses of the first order and second order hierarchical structures were much lower than that of the zeroth order hierarchical structure; stress was higher for second order at high rate.

Declaration of conflicting interests

Authors declare that no conflicting interests affected this research. Authors declare that no conflicting interests affected the objective presentation and description of results.

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