MSE 541 *Quiz 2*

Given: $\frac{E}{E_s} = C_1[\frac{\rho}{\rho_s}]$, $\frac{E}{E_s} = C_2[\frac{\rho}{\rho_s}]^2$, $\frac{E}{E_s} = C_3[\frac{\rho}{\rho_s}]^3$, $\nu_{31} = \nu_s$, $\nu_{21}^{regular} = 1$, $\nu_{iso} = \frac{1}{3}$, $\frac{\sigma_e^{regular}}{E_s} = 5.2[\frac{t}{L}]^3$, $\frac{\sigma_e^f}{E_s} = 0.05[\frac{\rho}{\rho_s}]^2$, $\sigma_{ij} = C_{ijkl}\epsilon_{kl}$, $\epsilon_{ij} = J_{ijkl}\sigma_{kl}$, $G_c = G_1V_1 + G_2V_2$, $J_c = J_1V_1 + J_2V_2$, $\delta \propto \frac{PL^3}{E_sI}$, $C_{1111} = E\frac{1-\nu}{(1+\nu)(1-2\nu)}$, $E = 2G(1+\nu)$, $\nu = \frac{3B-2G}{6B+2G}$, $A_c = \pi r^2$, $v_s = \frac{4}{3}\pi r^3$, $\epsilon_{ij} = J_{ijkl}\sigma_{kl} + d_{kij}\mathcal{E}_k + \alpha_{ij}\Delta T$, Epoxy resin, $E = 3.6$ GPa, $\sigma_{ult} = 90$ MPa, $\rho = 1.25$ g/cm^3; Boron fiber, $E = 400$ GPa, $\sigma_{ult} = 3.5$ GPa. Boron-epoxy unidirectional composite, $E_L = 210$ GPa, $E_T = 21$ GPa, $G_{LT} = 7$ GPa, $\sigma_{ult}^L = 2.6$ GPa, $\rho = 2.0$ g/cm^3. Graphite-epoxy unidirectional composite, $E_L = 160$ GPa, $E_T = 11$ GPa, $G_{LT} = 6.4$ GPa, $\sigma_{ult}^L = 1.72$ GPa, $\sigma_{ult}^T = 42$ MPa, $\tau_{ult} = 95$ MPa, $\rho = 1.6$ g/cm^3.

1
2
3
4
G

Solve **three** problems only and state which three. Scale begins at one point. **Show all logic. Enjoy!**

1. (33 points, 3 each) Define the following.
(a) laminate (b) stacking sequence (c) cross-ply laminate (d) HOBE block (e) cellular solid (f) relative density (g) porosity (h) rib re-orientation region (i) plateau region (j) tetrakaidecahedron (k) rib buckling

2. (33 points, 11 each) Consider foam made of aluminum.
(a) What are the true density and the relative density values of aluminum open-cell foam with the same stiffness (specifically, Young's modulus) as the polymer PMMA (solid, not foam)? Solid aluminum has a Young's modulus of 70 GPa and a density 2.7 g/cc; solid PMMA has a Young's modulus 3 GPa and a density 1.2 g/cc. Is the aluminum foam more dense or less dense than solid PMMA? Assume that the equation for modulus is valid over the full range of density.
(b) What is the relative density of an aluminum cellular lattice with $E = 3$ GPa based on the octet structure? Assume stretch dominated behavior such that $\frac{E}{E_s} = \frac{1}{6}[\frac{\rho}{\rho_s}]$. Briefly comment on the implications of your result.
(c) Commercial aluminum foam can be obtained with $\frac{\rho}{\rho_s} = 0.03$. Determine the Young's modulus E of this foam, and compare with the modulus of an octet lattice of the same density. What are the implications?

3. (33 points, 11 each) Consider the sole of a running shoe. Suppose the load supporting portion has an average values of thickness $h = 2$ cm, width $b = 8$ cm and length $c = 23$ cm. A person of mass 70 kg uses the shoe. The sole is made of material with elastic modulus $E = 1$ MPa.
(a) If the person stands on one foot, what is the compressive strain in the shoe sole? If the person runs so that peak force is twice the body weight, applied in the heel region covering one third the sole area, what is the compressive strain in the heel region of the sole? Do you think the sole has an effective cushioning effect? How could the cushioning be made better?
(b) The density of the sole material is about 0.4 grams per cubic centimeter. The solid material comprising the foam is a polymer with density 1 gram per cubic centimeter. What is the stiffness (Young's modulus) of the solid used to make this foam?
(c) Suppose the designer desires a lighter sole and suggests a foam with density of about 0.04 grams per cubic centimeter. Could this be made with a rubbery polymer as the solid phase? Explain, using quantitative arguments.

4. (33 points, 11 each) Consider structural elements within cellular solids. In the following, do not determine the proportionality constant.
(a) Show that for plate elements of width and length L and thickness t, $[\frac{\rho}{\rho_s}] = C_0\frac{t}{L}$.
(b) Show that for honeycomb compressed out of plane, the Young's modulus is given by $\frac{E}{E_s} = [\frac{\rho}{\rho_s}]$.
(c) Show that for honeycomb compressed in its plane, the Young's modulus is given by $\frac{E}{E_s} = C_3[\frac{\rho}{\rho_s}]^3$.