MSE 541 *Quiz 1*

Given: $\sigma_{ij} = C_{ijkl}\epsilon_{kl}$, $\epsilon_{ij} = J_{ijkl}\sigma_{kl}$, $G_c = G_1V_1 + G_2V_2$, $J_c = J_1V_1 + J_2V_2$, $G_L = G_2 + \frac{V_1}{\frac{1}{G_1-G_2}+\frac{6(K_2+2G_2)V_2}{5(3K_2+4G_2)G_2}}$, $K_L = K_2 + \frac{V_1(K_1-K_2)(3K_2+4G_2)}{(3K_2+4G_2)+3V_2(K_1-K_2)}$, $C_{1111} = E\frac{1-\nu}{(1+\nu)(1-2\nu)}$, $E = 2G(1+\nu)$, $\nu = \frac{3B-2G}{6B+2G}$, $A_c = \pi r^2$, $v_s = \frac{4}{3}\pi r^3$, $\epsilon_{ij} = J_{ijkl}\sigma_{kl} + d_{kij}\mathcal{E}_k + \alpha_{ij}\Delta T$, Epoxy resin, E = 3.6 GPa, σ_{ult} = 90 MPa, ρ = 1.25 g/cm^3; Boron fiber, E = 400 GPa, σ_{ult} = 3.5 GPa. Boron-epoxy unidirectional composite, E_L = 210 GPa, E_T = 21 GPa, G_{LT} = 7 GPa, σ^L_{ult} = 2.6 GPa, ρ = 2.0 g/cm^3. Graphite-epoxy unidirectional composite, E_L = 160 GPa, E_T = 11 GPa, G_{LT} = 6.4 GPa, σ^L_{ult} = 1.72 GPa, σ^T_{ult} = 42 MPa, τ_{ult} = 95 MPa, ρ = 1.6 g/cm^3, $\mathcal{D}_i = d_{ijk}\sigma_{jk} + K_{ij}\mathcal{E}_j + p_i\Delta T$.

1
2
3
4
G

Solve **three** problems only and state which three. Scale begins at one point. **Show all logic**. Enjoy!

1. (33 points, 3 each) Define the following, using as appropriate, a sentence or two, an equation or a diagram. For symmetry classes state the number of independent elastic constants. For materials, briefly relate the structure to the application. For effects, state an application.

(a) thermal expansion effect (b) asphalt (c) hexagonal (d) piezoelectric effect (e) orthotropic (f) isotropic bound formulae (g) cubic (h) pyroelectric effect (i) isotropic (j) carbon black filler (k) dental composite

2. (33 points, 11 each) Consider a Reuss composite in three dimensions.

(a) Sketch the 3-D structure. Show an applied compressive stress along the z axis.

(b) Determine the composite stiffness for compression along the z axis in terms of the constituent stiffnesses and volume fractions. Make simplifying assumptions if needed; state explicitly any assumptions made.

(c) If Poisson's ratio of each constituent is zero, determine *all* of the elastic constants of the composite. Show them as components of the C matrix.

3. (33 points, 11 each) Consider a material that may be anisotropic.

(a) Compressive strain ϵ_{zz} is applied in the z (or 3) direction to the ends of a sample. There is no strain in the other directions and there is no strain upon the lateral surfaces. Show that the stress-strain relation in the z direction is $\frac{\sigma_{zz}}{\epsilon_{zz}} = C_{3333}$. Hint: begin with Hooke's law in the modulus formulation.

(b) How might such a state of stress and strain be obtained experimentally?

(c) What is the physical meaning of C_{1313}? If the material is isotropic, is it true that $C_{1313} = C_{2323}$? What about $C_{1313} = C_{1323}$? Why?

4. (33 points, 11 each) Consider a three dimensional composite with identical spherical inclusions of radius r. Suppose during processing each inclusion is coated with a layer of matrix $r/10$ thick. The inclusions are then packed in a cubic array, with matrix in the intervening space.

(a) What is the volume fraction of inclusions?

(b) How does this composite differ in properties from one in which particles are in contact?

(c) What is the symmetry of the composite? How many elastic constants are there?