Lumped negative stiffness damper for absorption of flexural waves in a rod

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Abstract

A damper based on negative stiffness from column tilt buckling was used to achieve enhanced mechanical damping of bending vibration of a rod. Damping increased by a factor of three, via a damper column that was only about 0.2% of the mass of the aluminum alloy rod to be damped.

1 Introduction

In many applications, mechanical damping is desired to reduce noise of machinery, reduce vibration exposure of occupants and equipment in vehicles, and limit acceleration response in structures subjected to harmonic loading. Structural metals such as steel, brass and aluminum alloys exhibit very low damping $tan\delta$ of 10^{-3} or less (steel: 10^{-4} to 10^{-3}; brass: 10^{-4}; aluminum alloys: $< 10^{-5}$) [1] with δ as the phase angle between stress and strain sinusoids. Damping of structures can be achieved by adding layers of high-damping materials, typically polymers, or by external lumped dampers that may contain a viscous device. Practical polymeric damping layers exhibit peak values in damping $tan\delta$ from 0.1 to 1 or more. Polymers are much less stiff than structural metals so the strain energy density associated with damping is comparatively small. The figure of merit for damping layers is $Etan\delta$ with E as Young's modulus. Polymer layers exhibit high damping only over a narrow range of temperatures near the glass transition region so they provide limited performance if a structure is intended to function over a wide temperature range. The maximum figure of merit for polymer layers at the peak is typically $Etan\delta < 0.6$ GPa. Polymer layers also have the limitation that they decompose at high temperature.

High damping metals [2] [3] [4] are also used; they are usable over a wider temperature range than polymers. Moreover, they are stiff and can contribute to the structural rigidity of the structure to be damped. Copper-manganese alloys have been used to reduce vibration and noise emission in various applications such as in naval ship propellers [5]. Cu-Mn alloys are, however, nonlinear. They provide high damping only if the strain amplitude is sufficiently large. Zinc-aluminum alloys [6] have also been examined as high damping metals but they do not damp well at acoustic frequencies. Shape memory metals [7] and magnesium alloys [4] are also nonlinear; they can exhibit

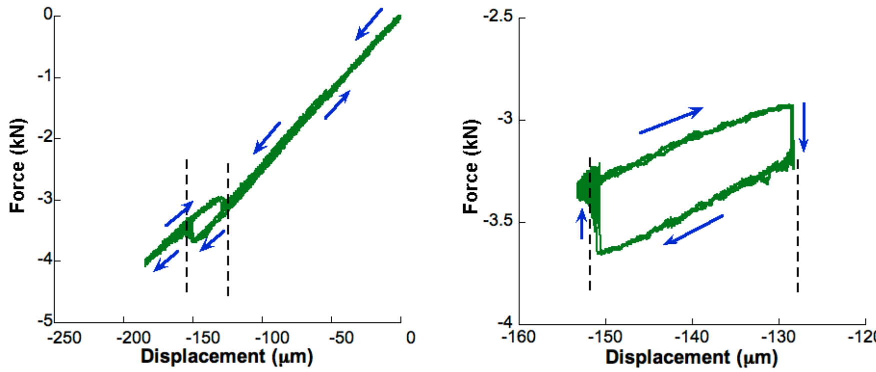

Figure 1: Hysteresis and negative incremental stiffness of a constrained steel flat end rod in buckling, replotted using data from [16]. Left: larger deformation range. If the range is restricted (vertical dash lines) by a preload, the effective damping is higher, as shown on Right: smaller deformation range.

high damping over a range of temperature if the strain amplitude is sufficiently large. High damping combined with high stiffness in the linear regime has been attained in designed metal matrix composite materials [8, 9] that could be used as damping layers.

High damping can also be achieved using structural elements (buckled flexible tubes) with negative stiffness [10] or composite materials with inclusions of negative stiffness [11] [12] [13]. Damping in these composites was evaluated in the regime of small strain amplitude. Negative stiffness structures and materials in isolation are unstable. If they are appropriately constrained, then they can generate high effective damping. Structures based on lateral force on sigmoid shaped beams or axial force on buckled flexible tubes provide a negative stiffness region by which high structural damping is attained [10]. Such damping can be achieved at small deformations in the linear regime or via nonlinear hysteresis via nonlinear snap through effects. Negative stiffness structures with sigmoid beams including many more recent ones [15] are compliant. For many applications, both high stiffness and high damping are required.

Lumped dampers that provide high stiffness and high damping and which utilize negative stiffness elements have been designed and demonstrated experimentally [16]. These devices consist of a frame and flat end columns each of which is initially in contact with flat surfaces. As the frame is compressed, a column buckles and effectively snaps from fixed end conditions (via flat end contact) to pinned end conditions (via tilt to edge contact). This change in boundary conditions causes a hysteresis that results in mechanical damping. The dissipation of energy is shown as the area within hysteresis loops for a steel flat end rod in buckling, (figure 1). The near vertical portions of the curve represent negative incremental stiffness because the slope of the load-deformation curve is reversed. The frame provides constraint that is required to stabilize the system in the negative stiffness regime. These experiments were conducted at sub-audio frequency, 0.5 Hz. The combination of stiffness and damping can substantially exceed that of known materials. Specifically, a maximum stiffness-damping product of approximately 200 GPa was achieved at 0.5 Hz in dampers based on stainless steel columns [16], compared with less than 0.6 GPa for polymer damping layers at the optimal temperature and frequency. The effect in the buckled columns is nonlinear. To attain the peak performance the strain range and the pre-strain must be chosen appropriately. Polymer column dampers are less sensitive to strain tuning and can be operated in the linear regime [14]. Considerable subsequent research in this area has been done [15] [17].

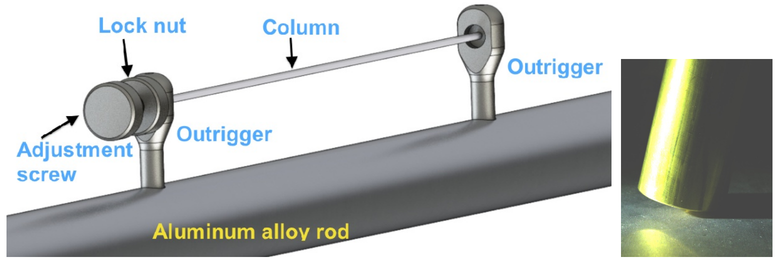

Figure 2: Diagram of the damper and aluminum alloy rod (left). End tilt of a column (right)

Previous research with negative stiffness hysteresis dampers has focused on damping axial displacements, particularly at low frequency in the quasi-static regime. In this paper, flexural waves in a rod are damped with a lumped damper based on the hysteresis associated with column buckling.

2 Methods

The object to be damped is a rod of 6061 aluminum alloy, 25.4 mm (1") in diameter and 1.83 m (6') long. The rod is supported by vertical strings about 0.9 m long, 0.4 m from each end. The flexural damper consists of two outriggers and a column. The outriggers are screwed into 1/4-28 tapped holes in the rod and spaced 200 mm apart. One of the outriggers is adjustable by means at an adjustment screw with a locknut, which have threads with a pitch of 56 threads per inch for fine adjustment. The outriggers, the adjustment screw and locknut were machined from AISI 1045 medium carbon steel, which is commonly used as screw stock. The column is fit between the outriggers each of which has a flat surface upon which the damper column can tilt. The damper column is a 5/64 inch (1.98 mm) diameter rod of AISI 304 stainless steel and has a mass of 4.79 grams. The rod to be damped has a mass of 2.57 kg. The damper column is 1/537 (about 0.2%) of the mass of the rod to be damped. The damper column offset from the rod surface is about 24 mm. A drawing of the apparatus is shown in figure 2. A detailed view of column end tilt during buckling is shown on the right.

The column damper was designed as follows. The column diameter was chosen to be a commercially available size of stainless steel rod. The limiting failure mode of the system was determined to be yielding of the outrigger. For the column size chosen, the calculated stress in the outrigger was about half the yield stress. A larger diameter column would generate sufficient force during buckling to require a different end support. From Kalathur et al. [18], the column buckles at a strain close to what is predicted by Euler buckling theory for fixed ends and the load-deformation relationship is linear until buckling. Therefore, the maximum force on the outrigger is the fixed end Euler buckling force. A design study was conducted with outrigger length and column length as design variables. The maximum bending stress in the outrigger was the output of the design analysis. The corresponding load was used to choose column elements that would not generate enough force to permanently deform the outrigger supports.

Initially, there were two dampers on the column, mounted in opposition in the same tapped hole that extends laterally through the rod. However, it was found that the outriggers rotated in their threads in this configuration. To prevent the outriggers from moving in this way, the outriggers on the other side were screwed tightly against the the base of the outriggers in use, locking them in place. To prevent translation of the column upon its flat contact surface, a layer of cellophane tape was applied to the surface; the column pierced the tape and was allowed to rock in place.

To monitor the vibration in the rod, three Vishay Micro-Measurements general purpose strain gages (CEA-06-062WT-120) were mounted on the rod in alignment with the outriggers. The strain gage wires hung vertically from the rod about 0.4 m. Each group of three wires had a mass per length of 67 mg / cm. A Vishay Micro-Measurements 2100 signal conditioner with 2120A bridges was used. A Tektronix DPO 3014 digital oscilloscope was used to monitor and record the output of the signal conditioner.

To excite the structure, the end of the rod was struck transversely in a horizontal direction with a hammer and the transient bending vibration was recorded. Repeated impacts were done until the desired range of strain was obtained. Tests were done at ambient temperature, 21 °C. Damping was determined by fitting an exponential in time to a group of points representing peaks of the damped sinusoid in a selected region. The damping of the system was inferred from the decay of resonant decay using equation (1) in which T is the period of the oscillation and $t_{1/e}$ is the time for the amplitude of the oscillation to decay to $1/e$ of its initial value [1]. This assumes linearity; if the system is nonlinear, it is an effective tan δ.

$$\tan\delta = \frac{1}{\pi}\frac{T}{t_{1/e}} \tag{1}$$

The damping of higher modes was found by recording the first 200 ms of the response at a high sampling density. The sampling rate was 50 kHz. A mode at 5 kHz was detected and its decay with time was found approximately by measuring the magnitude of sinusoids with periods corresponding to 5 kHz. An exponential curve fit was applied to magnitudes and the times they occurred and this was converted to damping as described above.

The adjustment screw was tuned by tightening it until the column began to buckle. This was observed by sighting longitudinally along the column. Buckling occurs when the adjustment screw is advanced about $^3/_8$ of a turn in from initial contact between the screw and column. This gives an initial strain of 8.5×10^{-4} in the column. To achieve high damping in the system, careful tuning of the adjustment screw is required. An adjustment of about 5 degrees (a change in strain in the column of 6.3×10^{-5}) makes a significant difference in the resulting damping of the system. This strain was calculated from screw motion.

3 Results and discussion

Impact on the aluminum alloy rod generated multiple modes of vibration. The fundamental bending mode frequency was 33 Hz. The theoretical natural frequency f_n of the rod, modeled as a free-free beam, was found to be 34.2 Hz, where $f_n = \frac{22.4}{2\pi}\sqrt{\frac{EI}{\rho A\ell^4}}$, $A = \pi r^2$, and $I = \frac{\pi r^4}{4}$ [19]. Input parameters were $E = 69$ GPa, $\rho = 2700$ kg/m^3, length $\ell = 1.83$ m (6ft) and $r = 0.0127$ m.

Damping of the rod without damping columns was exponential in nature as shown in in figure 3. All plots are from the output of the strain gage located between the outriggers. Output from the other strain gages was similar in both shape and amplitude. The exponential curve fit is to the initial part of the transient. Without the column, the damping $tan\delta$ of the 33 Hz mode of the system is 8.64×10^{-4}. This is substantially larger than the reported intrinsic damping of this alloy. Aluminum alloy 6061 exhibits tan $\delta{\approx}3.6\times10^{-6}$ in torsional vibration at 1 kHz, for a strain less than 10^{-6} at room temperature [20]. The difference is attributed to the following. The intrinsic damping of the aluminum alloy is likely to be frequency dependent; no published results are know at low frequency at the low end of the audio range. Bending vibrations are damped by viscosity of the airas well as by radiation of sound into the air and into attachments such as the strain gage wires and support strings used in this study. Also, dislocation damping via the strain-dependent

Granato Lücke mechanism [21] may contribute at the strain levels used in this study. This is not a problem because the focus of this study is to evaluate the efficacy of damping interventions in increasing the effective damping; the intrinsic tan δ of the aluminum alloy is not of interest.

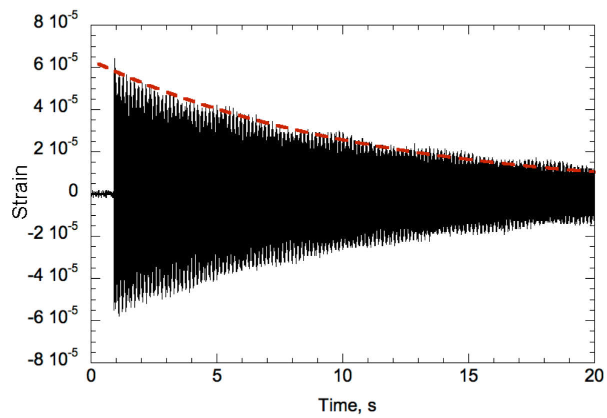

Figure 3: Transient response of aluminum alloy rod with no damping column. The effective damping $tan\delta$ is 8.64×10^{-4}.

Addition of one column increased the damping (effective tan δ) of the initial transient to 2.50×10^{-3}, an increase of a factor of about 2.9, or an additive damping of 1.64×10^{-3}. This is substantial in view of the small mass of the column damper in comparison with the rod to be damped. Response is shown in figure 4. A beat phenomenon with a period of about 10 seconds is evident after the initial transient. The beats are a result of two closely spaced frequencies in the mode structure. The fundamental frequency becomes split because the damping column introduces a slightly greater stiffness for bending in the direction of the column assembly than in the orthogonal direction.

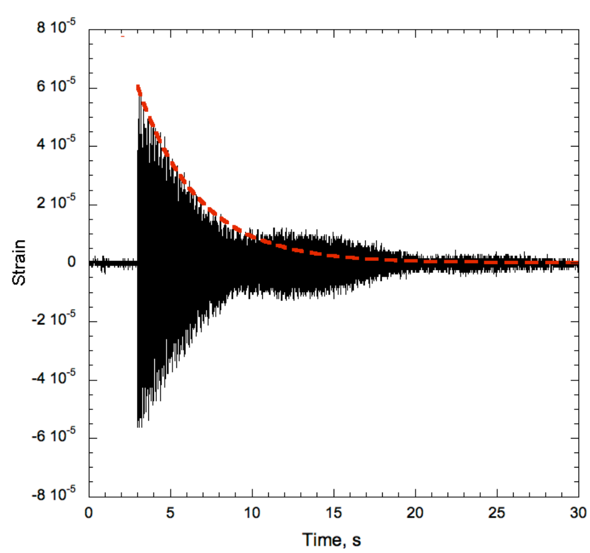

Figure 4: Transient response of aluminum alloy rod with one column. The exponential curve fit is to the initial part of the transient. The initial damping of the 33 Hz mode of the system is 2.50×10^{-3}.

Addition of two columns increased the damping to 3.6×10^{-3}, an increase of a factor of about 4.2

compared with the rod without damping columns. Compared with one column this is an additive damping of 1.1×10^{-3}. Compared with no column this is an additive damping of 2.7×10^{-3}. Response is shown in figure 5. A more pronounced beat phenomenon with a period of about 10 seconds is evident after the initial transient. As in the case of one column above, this is due to a superposition of two low frequency modes with similar frequencies. The cause is attributed to asymmetry in the load-deformation characteristic from the attached dampers. The beats make it difficult to obtain an accurate value of the damping in the low amplitude regime after the initial transient.

Column buckling requires a threshold deformation (figure 1). The hysteresis due to the negative stiffness of the column occurs over a range of axial deformation. Therefore the damping will enhanced until the displacement is no longer large enough to activate the buckling column. Then the damping is expected to decrease. Measurement of this reduced damping was, however, problematical as a result of the beat phenomenon and also the ratio of signal to noise when the strain amplitude is low.

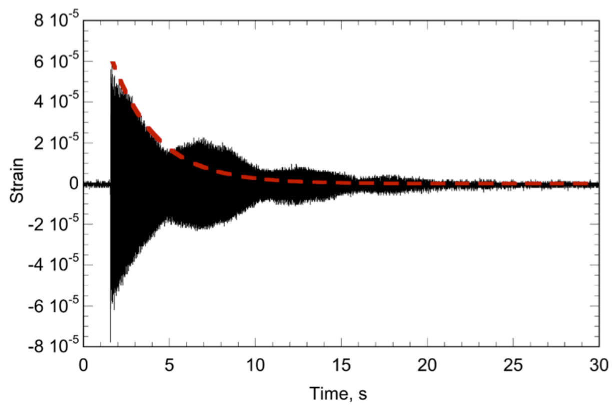

Figure 5: With two columns side by side, the initial damping (effective tan δ) of the 33 Hz mode of the system is 3.60×10^{-3}. The exponential curve fit is to the initial part of the transient.

Higher modes in the kHz regime were clearly audible; they damped more quickly than the fundamental mode. Figure 6 shows the damping of a 5 kHz mode compared to the damping of the fundamental mode for all cases: no damping column, one damping column, two damping columns.

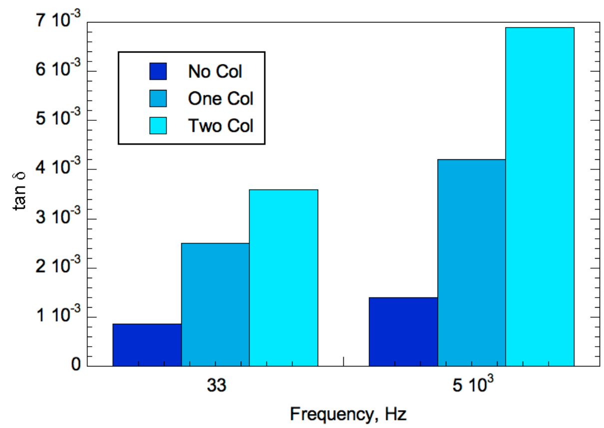

Figure 6: The damping (effective tan δ) of the 33 Hz mode and of a 5 kHz mode of the system for all cases: no damping column, one damping column, two damping columns.

Dampers based on column buckling are highly effective in relation to their weight [16]. The reason is that the figure of merit $E tan \delta$ of column based dampers is much greater than that of negative stiffness dampers based on sigmoid shaped beams. Referring to figure 1, the effective damping for an appropriate deformation amplitude of a column damper alone is about 0.88. Effective damping refers to specific damping capacity divided by 2π; it reduces to tan δ in the linear regime. The present damper had only 1/537 of the mass of the rod to be damped. Based on the mass ratio, one might expect a maximum additive damping of $0.88/537 = 1.6 \times 10^{-3}$. Because tuning of the damper was based on a screw adjustment, and impacts were of various magnitude, it is not to be expected that the maximum be attained in these experiments. The observed additive damping for one column was 1.64×10^{-3}. The offset of the column damper with respect to the rod axis will provide some advantage.

The damper could be refined by eliminating the outrigger and placing damping columns in machined notches in the aluminum alloy rod. Thin steel contact plates are required in such a design to prevent indentation of the aluminum. Tuning could then be done electrically by piezoelectric actuators. Such an approach would confer some weight reduction via elimination of the outrigger; also the potential for a simpler mechanical system. Use of more dampers has the potential to increase the overall damping if required.

Further refinement may include compensation for any mismatch in thermal expansion, so contact in the damper is maintained through temperature changes.

The present damper is based on snap buckling of steel which is substantially insensitive to temperature. By contrast, polymer damping layer materials typically perform well over a restricted temperature range of 10 to 20°C. This is because polymer dampers are based on the high damping that occurs near the glass-rubber transition in polymers. Damping materials based on negative stiffness associated with phase transformations [12] [13] are also highly temperature sensitive. Lumped dampers based on snap buckling do not require special materials such as the high damping metals discussed above or high damping composites. For example, ZnAl-SiC composites displayed a high figure of merit over a broad temperature range [22] but dampers based on buckled steel columns may be more practical because they do not require special materials. The additive damping effect is particularly beneficial if the baseline damping is lower as is the case in satellites and spacecraft which operate under vacuum.

4 Conclusion

A damper based on negative stiffness from column tilt buckling was used to achieve mechanical damping enhanced by about a factor of three of bending vibration of a rod. The damper column was about 0.2% of the mass of the aluminum alloy rod to be damped.

5 Acknowledgements

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