EMA 630 Viscoelastic Solids
Midterm Quiz

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Given: $e_{rr} = \partial u_r/\partial r$; $e_{\phi\phi} = (\partial u_\phi/\partial\phi + u_r)/r$; $e_{zz} = \partial u_z/\partial z$; $e_{\phi z} = [\partial u_\phi/\partial z + (1/r)\,\partial u_z/\partial\phi]/2$ $e_{zr} = [\partial u_z/\partial r + \partial u_r/\partial z]/2$; $e_{r\phi} = [\partial u_\phi/\partial r + (1/r)\,\partial u_r/\partial\phi - u_\phi/r]/2$, $\sigma_{ij,j} + F_i = \rho a_i$; $\sigma = E\varepsilon$; $\sin(\theta \pm \varphi) = \sin\theta\cos\varphi \pm \cos\theta\sin\varphi$. $J^* = 1/E^*$.

$$J''(\omega) = \frac{2\omega}{\pi}\left[\wp\int_0^\infty \frac{J'(\varpi)-J'(\infty)}{\varpi^2-\omega^2}d\varpi\right] \quad J'(\omega) - J'(\infty) = \frac{2}{\pi}\left[\wp\int_0^\infty \frac{\varpi J''(\varpi)}{\varpi^2-\omega^2}d\varpi\right],\ \sigma(t) = \int_0^t E(t-\tau)\frac{d\varepsilon(\tau)}{d\tau}d\tau$$

$$\mathcal{L}[f(t)] \equiv F(s) \equiv \int_0^\infty f(t)e^{-st}dt,\ \nu = \frac{3B-2G}{6B+2G} = \frac{1}{2} - \frac{E}{6B},\ E = 2G[1+\nu],\ x(t) \approx x_0\, e^{-(\omega t/2)\tan\delta}\sin\omega t.\ \mathcal{L}\left[\frac{df(t)}{dt}\right] = s\mathcal{L}[f(t)] - f(0),\ E = 2G(1+\nu)$$

$$\int \frac{x}{(x^2-a^2)^2}dx = -\frac{1}{2(x^2-a^2)}.\ \text{ctn}\,x \cong \text{ctn}\,[\pi/2] + [-\csc^2(\pi/2)][x-\pi/2] + \ldots \quad \mathcal{L}[e^{-at}] = \frac{1}{s+a},\ \mathcal{L}[1] = \frac{1}{s},\ \mathcal{L}[\mathbf{H}(t)] = \frac{1}{s},$$

$$\mathcal{L}[t] = \frac{1}{s^2},\ \mathcal{L}[\mathbf{H}(t-a)] = e^{-as}/s,\ \mathcal{L}[\delta(t-a)] = e^{-as},\ \mathcal{L}[t^n e^{-at}] = \frac{n!}{(s+a)^{n+1}},\ \mathcal{L}\left[\frac{t^{n-1}e^{at}}{(n-1)!}\right] = \frac{1}{(s-a)^n},\ R = 1.98\ \text{cal/moleK}$$

$$\mathcal{L}\left[\int_0^t f(t-\xi)g(\xi)\,d\xi\right] = \mathcal{L}[f(t)]\,\mathcal{L}[g(t)],\ \mathcal{L}[\sin(at)e^{-bt}] = \frac{a}{[(s+b)^2+a^2]},\ \mathcal{L}\left[\frac{[be^{bt} - ae^{at}]}{(b-a)}\right] = \frac{s}{(s-a)(s-b)}\ \text{for } a\neq b,\ \ln\frac{\nu_2}{\nu_1} = \frac{U}{R}\left\{\frac{1}{T_1} - \frac{1}{T_2}\right\}.$$

Solve **four** problems only and state which four. Show all **logic** and state assumptions!!

1 (25 pts) Define the following. One sentence or a diagram or an equation should suffice.

(a) attenuation
(b) tan δ
(c) resonance
(d) shift factor
(e) stretched exponential
(f) Boltzmann superposition principle
(g) spectrum of relaxation times
(h) Debye peak

2 (a) (10 pts) Show that for a linearly viscoelastic material, $J' = \frac{1}{E'}\frac{1}{1+\tan^2\delta}$

(b) (10 pts) What is the physical interpretation of $\frac{1}{J'} > E'$ in the context of a stress-strain diagram?

(c) (5 pts) How are J' and E' related for an elastic material?

3 (a) (5 pts) Draw the stress strain curve for a linearly viscoelastic solid under sinusoidal strain $\varepsilon(t) = B\sin\omega t$.

(b) (5 pts) Suppose the stress is $\sigma = D\sin(\omega t + \delta)$. What is the meaning of δ?

(c) (5 pts) Find the slope of the line from the origin to the point of maximum stress. Hint: write strain in terms of stress and let the stress assume its maximum value; start with $\sigma(t) = \sigma_{max}\sin(\omega t)$.

(d) (5 pts) Show $\sin\delta = A/B$. A is the intercept on strain axis. Hint: let $\omega t = -\delta$ in equation in (b) for the stress.

(e) (5 pts) Draw a stress strain curve for a nonlinearly viscoelastic material under sinusoidal strain.

4 The end deflection u of an elastic cantilever beam of length L, Young's modulus E and cross sectional area moment of inertia I is given as $u = 2FL^3/6EI$, with F as the applied force.
Suppose now the beam is linearly viscoelastic and that any needed viscoelastic properties are known. If the force has a time dependence F(t), determine the time dependence u(t) of the deflection.

5 Consider $E(t) = A + Be^{-tb}$ with A, B, b as constants.
(a) (10 pts) If a relaxation experiment were conducted on your earplug material, do you expect it would follow the above equation? Explain why or why not.
(b) (15 pts) Suppose a material with the given E(t) is subject to step strain $\varepsilon(t) = [\varepsilon_o + \varepsilon_1 e^{-at}]\mathbf{H}(t)$, with a as a constant and ε_o, ε_1 as constants. Hint: assume strain starts just after zero so no surface terms.
Determine the stress σ(t).