Resonant ultrasound spectroscopy [RUS] R. Lakes, University of Wisconsin

RUS allows one to determine modulus [1] and damping [1,2] of cubic, sphere, or short cylinder specimens without gluing or alignment. RUS can be done with standard ultrasonic transducers and available electronics or by commercial systems. Shear transducers [3] give a much stronger signal for the fundamental mode than the usual longitudinal transducers. $LiNbO_3$ is good as a high temperature transducer.

Method. Drive one piezoelectric transducer with a function generator. Observe the output of the other transducer on an oscilloscope or a lock in amplifier. The specimen is a cube in contact with the transducers at its corners or a short cylinder in contact at the edges of its ends. Cable length from output should be less than 1 m. Use the screw jack to adjust the height of the lower transducer until the specimen is held in place. Use micrometer z-axis stage for fine adjustment to minimize contact force to reduce parasitic damping and frequency shift.

Raise the frequency (manually or by a scan) until the first resonance is seen. For a cube or short cylinder (of length equal to its diameter) this is a pure torsion mode for which it is easy to calculate the modulus. Resonance sharpness and peak height inverse to damping. If necessary, amplify function generator or sensor transducer output to get a stronger signal. **Scan** by modulating the function generator frequency. Observe the response on a digital scope.

Identify the **lowest mode**, the fundamental, via the known mode structure for the specimen shape (for isotropic solid). Higher modes begin a factor 1.26 (cylinders) to 1.35 (cubes) in frequency above the fundamental, so if there are no modes a factor two in frequency below a candidate mode, it is the fundamental. From this mode, get the **shear modulus G**. Use numerically generated plots of mode frequency vs. Poisson's ratio to obtain **Poisson's ratio** for isotropic material. The following formulae are for isotropic material unless otherwise stated.

Cube resonance [4], fundamental torsion resonant frequency $f_1 = \frac{\sqrt{2}}{\pi L}\sqrt{\frac{G}{\rho}}$

L = side length, ρ = density. $\sqrt{2}/\pi \approx 0.45$. The lowest Mindlin Lamé [5] shear mode in the cube is a factor 1.57 up from the fundamental. For a hexagonal or isotropic cube $f_m = (m/L)(\sqrt{2}/2)\sqrt{C_{66}/\rho}$; $C_{66} = G$. m is a positive integer. For a cubic or tetragonal cube replace $2C_{66}$ with $(C_{11} - C_{12})$. These modes (often weak), with their integer spacing, allow analytical determination of shear moduli. For a 1 cm brass cube the fundamental frequency $f_1 \approx 91$ kHz; for a 1 cm PMMA cube $f_1 \approx 61$ kHz. The Demarest plot normalizes frequency to $f_{norm} = (1/\pi L)\sqrt{G/\rho}$.

Cylinder resonance, torsion, fundamental frequency $f_1 = \frac{1}{2L}\sqrt{\frac{G}{\rho}}$. Bessel, $f_b = \frac{2\beta_p a}{\pi}\frac{1}{4a}\sqrt{\frac{G}{\rho}}$.

L = cylinder length. There is no restriction on the length for this to be true, but for a long cylinder the first mode becomes a bending mode for which calculation is much more complicated. Use a short cylinder with length equal to diameter. [6] Torsion modes of the cylinder are in ratio 1,2,3,4 so $f_m = (m/2L)\sqrt{G/\rho}$ with m, integer mode number. Bessel modes [7] are given by $u_\theta = BJ_1(\beta r)\, e^{i(\gamma z - \omega\tau)}$; (a = R is radius); $(\beta_p a)^2 = (\omega a/V_s)^2 - (\gamma_p a)^2$, with $\beta_p a = 5.136$, 8.417 .. For the lowest group of these modes the outside and inside of the cylinder rotate in opposite directions with no z dependence ($\gamma_p = 0$). For L = 2R, the lowest of these is a factor 3.27 higher than the torsion fundamental frequency. For a cylinder, align the edges with the shear transducers to achieve torsion resonance[3]. The shear direction is along the direction of the BNC cable connector. If the resonant mode is torsion, the signal vanishes if the cylinder is rotated 90° about the axis formed by the points of contact.

Sphere of radius R, fundamental torsion resonant frequency $f_1 = (2.48/2\pi R)\sqrt{G/\rho}$.

To get the **damping** use the resonant full width Δf at half maximum ; $\mathbf{Q^{-1} \approx \tan\delta \approx \Delta f/(\sqrt{3} f_o)}$.

If $\tan\delta < 10^{-3}$, minimize the contact force to reduce parasitic damping from transducer contact, which contributes error, noticeable in low damping materials. Parasitic damping is less in the higher modes.

For **anisotropic** or isotropic solids, all the elastic modulus elements can be obtained via a computer algorithm in which many resonance frequencies are input, with specimen dimensions and density. The mode peaks must be sharp for this to work. Slight anisotropy leads to a splitting of the modes found for isotropy. For a cylinder, the fundamental is not split.

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