for approximate and/or numerical models such as, for example, the finite element technique. For this reason, Table 1 is appended, giving vlaues of K_2 (correct to three decimal places) for various ϕ's and γ's.

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A Pathological Situation in Micropolar Elasticity

R. S. Lakes[1]

The case of zero coupling number N *in micropolar elasticity is considered. This situation has been examined by several investigators to simplify the analysis of micropolar materials. We show that the case of $N=0$ is pathological and present a physical example.*

Introduction

Micropolar elasticity is a continuum theory that allows degrees of freedom not present in classical elasticity. The extra degrees of freedom are thought to describe some aspects of the deformation of materials with microstructure. The constitutive equations for a linear, isotropic micropolar solid are [1]:

$$t_{kl} = \lambda e_{rr}\delta_{kl} + (2\mu+\kappa)e_{kl} + \kappa\, e_{klm}(r_m - \phi_m) \quad (1)$$

$$m_{kl} = \alpha\, \phi_{r,r}\delta_{kl} + \beta\, \phi_{k,l} + \gamma\, \phi_{l,k} \quad (2)$$

in which t is the asymmetric force stress, m is the couple stress, $e_{kl} = \frac{1}{2}(u_{k,l} + u_{l,k})$ is the small strain, u is the displacement, ϕ is the microrotation, and e_{klm} is the permutation symbol. $r_k = \frac{1}{2}e_{klm}u_{m,l}$ is the macrorotation, which is kinematically distinct from the microrotation. α, β, γ, and κ are micropolar elastic constants; when they vanish, classical elasticity is recovered as a special case.

Significance of κ

Following [2, 3] we define the coupling number N: $N^2 = \kappa/2(\mu+\kappa)$. N is dimensionless and its range of permitted values is [0, 1], based on energy considerations [2]. N is a measure of the intensity of coupling between the microrotation and macrorotation fields. When $N = 1$, these fields become perfectly coupled, so the microrotation ceases to be an independent kinematical degree of freedom. This special case corresponds to the indeterminate couple stress theory [4]. For this case, many of the effects predicted to occur in a micropolar solid, e.g., stiffening of thin rods in bending [3] and in torsion [5], and reduction of the stress concentration around a hole [2], are maximum.

Several authors have considered the special case $\kappa=0$, or equivalently, $N=0$. This special case may considerably simplify problems for which it is difficult to obtain an explicit solution [6] or may be used to explore limiting behavior of solutions of micropolar problems [7]. The solutions to some problems are observed to coincide with classical elasticity, if $N=0$, [2, 1].

A Torsion Problem

The problem of quasistatic torsion of a long circularly cylindrical rod of an isotropic, micropolar medium has been solved by Gauthier and Jahsman [5]. In this Note we explore solutions to this problem for the case of zero coupling number. In the general torsion problem, the displacement and microrotation fields are uniquely specified by the applied torque and by the assumption of translational symmetry in the axial direction. For the case of zero coupling number N we observe a different situation: a given value of applied torque can be realized in more than one way.

Specifically, the case of zero applied torque may be realized by either of the following displacement fields:

$$\left\{\begin{array}{l} u_r = 0 \\ u_\theta = C_1 rz \\ u_z = 0 \end{array}\right\} \quad \left\{\begin{array}{l} \phi_r = C_2(-\alpha/(2\alpha+\beta+\gamma))r \\ \phi_\theta = 0 \\ \phi_z = C_2 z \end{array}\right\} \quad (3)$$

in which $C_2 = -C_1[(R^2/2)(\mu/[(\beta+\gamma) + \alpha(1 - \alpha/(2\alpha+\beta+\gamma))])]$, R is the cylinder radius and C_1 is any number of dimension 1/length; and

$$\left\{\begin{array}{l} u_r = 0 \\ u_\theta = 0 \\ u_z = 0 \end{array}\right\} \quad \left\{\begin{array}{l} \phi_r = 0 \\ \phi_\theta = 0 \\ \phi_z = 0 \end{array}\right\} \quad (4)$$

In field (3), there arise opposing, nonzero states of force stress and couple stress that result in zero net torque upon the end surfaces. In keeping with the boundary conditions, there must be a prescribed self-equilibrated distribution of force stress and couple stress upon the end surfaces, or the displacement and microrotation must be prescribed on these surfaces. For the case $N\neq 0$, by contrast it is not possible to achieve the displacement and microrotation field (3) since the difference between the macrorotation and microrotation gives rise to an antisymmetric contribution to the force stress by virtue of (1) for $\kappa\neq 0$. This contribution makes it impossible to satisfy the equilibrium equations, therefore for $N\neq 0$, field (4) is the unique displacement and microrotation field associated with zero end torque (and zero end force).

For the special case $N=0$, the constitutive equations (1) and (2) may be used to show that field (3) satisfies the equilibrium equations and boundary conditions. Field (3) also must automatically satisfy the micropolar compatibility conditions [1] since the displacement-rotation field, rather than the strain field, has been prescribed. This may be verified by using the tensorial form of the compatibility conditions to obtain an expanded form for cylindrical coordinates. The expanded form given in [8] must be viewed with caution since this version has been specialized under the implicit assumption that the micropolar strain $\epsilon_{kl} = e_{kl} + e_{klm}(r_m - \phi_m)$ has no z dependence. This assumption does not apply to field (3), therefore the full compatibility conditions given in [1] must be considered. In field (3), the conventional strain e_{kl} is, however, independent of z and, with $N = 0$, the stresses are also independent of z.

Discussion and Conclusion

The case $N=0$ does not correspond to a failure of uniqueness in the usual sense, since different local boundary tractions are associated with the displacement and microrotation fields (3), (4). The total macroscopic load, however, is the same at the end, which may be arbitrarily far from the region of interest. The different stress distributions at the end in fields (3) and (4) are therefore equivalent in the sense of Saint-Venant. In this sense, the case $N=0$ is pathological. For some specific problems, the solution for $N=0$ is found to coincide with the solution for classical elasticity. This case is not equivalent to classical elasticity, however; the microstructural degrees of freedom remain. Classical elasticity is recovered as a special case of micropolar elasticity only if α, β, γ, and κ all vanish.

A physical example that exhibits some features of the situation just considered is as follows. Consider a long rod of a composite material made of parallel stiff fibers embedded in a compliant matrix. Fiber orientation is random, so macroscopic properties are isotropic. The spatial average of force per unit area upon fibers and matrix may be regarded as the force stress, and the corresponding spatial average of couple upon each individual fiber, per unit area, may be regarded as the couple stress. At each end of the rod, we may macroscopically twist the end by a given angle, and microscopically twist the end of each fiber in the opposite direction until the net end torque is zero. If the interface between fiber and matrix is perfectly lubricated, the effect of the end displacements and rotations may be expected to propagate an arbitrary distance down the rod. This situation is analogous to the continuum case $N=0$ considered in the foregoing. Significant end effects may also occur in classical elasticity of highly anisotropic materials [9]; by contrast the preceding example depends on micromechanical degrees of freedom rather than anisotropy.

Micropolar elasticity has been found to be useful in the interpretation of recent experiments upon solids with fibrous [10] and cellular [11] structures. Nonzero values of N were inferred from these experiments. No structures corresponding to the preceding physical example, however, were studied.

To conclude, the special case $N=0$ in micropolar elasticity is pathological in that specification of the macroscopic end load on a rod does not uniquely determine the displacement and microrotation fields far from the end. The dependence of these fields on remote, localized distributions of self-equilibrated load calls into question the applicability of Saint-Venant's principle for such solids.

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Effect of Debonding on the Stability of Fiber-Reinforced Composites

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1 Introduction

In this work we present some analytical solutions to a model problem in order to elucidate the effects of a preexisting debonding between fiber and matrix on the stability properties of reinforced composites.

Our analysis is motivated by some recent experimental [1] and theoretical investigations [2, 3] on delamination buckling of fiber-reinforced and laminated composites subjected to compressive loads along the reinforcing direction. Of particular interest is the effect of the debonded length on the critical buckling load which is investigated here for two extreme cases of the fiber-to-matrix relative stiffness. By ignoring fiber interaction effects and using a simple model for our problem, we are able to obtain closed-form analytical solutions.

More specifically, we consider a beam on two elastic type foundations. The effect of debonding in a certain region is modeled by the local exclusion of tensile lateral forces. Realistic buckling loads and eigenmodes have been found for all the cases considered and all critical loads were of the same order of magnitude as that for the fully bonded composite. Our results at this stage are preliminary in nature, but they indicate the possibility of solution of some more complicated but more realistic delamination buckling problems.

2 Fiber Buckling in a Soft Matrix

To study the effects of debonding in the fiber buckling of a soft matrix composite, the matrix material on each side of the fiber is idealized as an elastic foundation of modulus k. Two cases will be considered. In the first case the entire fiber is assumed to be debonded from the matrix material so that tensile stresses at the matrix-fiber interface are nowhere allowed. In the second case debonding will be assumed only in a finite zone of length $2a$.

2.1 Fully Debonded Fiber (Debonded Zone $-\infty \le x \le +\infty$). In the case of full debonding, every point of the fiber will be in contact with one of the two foundations (the one corresponding to a compressive distrbuted load). Consequently, the problem is equivalent to the buckling (under a lateral force P) of a continuous beam resting on only one elastic foundation of modulus k. The critical load for this case is well known (see, for example, Timoshenko and Gere [4]) and is

$$P_d = (4\,k\,EI)^{1/2} \tag{2.1}$$

where EI is the fiber's bending stiffness. For the fully bonded fiber, of course, both the elastic foundations on each side of the fiber will be active at buckling, and the total effective foundation stiffness will be $2k$. Therefore the corresponding lateral buckling load P_b should be given by

$$P_b = (8\,K\,EI)^{1/2} \tag{2.2}$$

For the partially bonded fiber to be analyzed subsequently, the corresponding critical load P_p is expected to satisfy $P_d \le P_p \le P_b$.

2.2 Partially Debonded Fiber (Debonded Zone $-a \le x \le a$). If $u(x)$ is the lateral displacement of the fiber due to

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