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GENERALIZED CONTINUUM MECHANICS OF COMPACT BONE:

A COMPARISON, COSSERAT ELASTICITY AND THE THEORY OF VOIDS

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Abstract. Experiments were designed and performed to discriminate between Cosserat (micropolar) elasticity and the theory of elastic materials with voids as generalized continuum models for wet and dry bone. In dry bone the Cosserat elastic effects seen in wet bone were largely suppressed. Nevertheless, it was not possible to isolate the contribution of the voids to the micromechanics of dry bone.

Introduction. Human compact bone exhibits a variety of microstructural features. Some features such as the osteons are large enough that a continuum representation of bone will need to incorporate additional degrees of freedom to describe the micro-motions of the microstructure. Two continuum theories may be considered for the description of compact bone: Cosserat or micropolar [1] elasticity and the theory of elastic materials with voids [2,3]. In Cosserat elasticity the additional kinematical degree of freedom is a local rotation of the microstructure, while in the void theory, it is a change in the solid volume fraction. Laminae in bovine bone [4] and presumably also osteons in human bone can exhibit relative motion, so one can attribute local rotations to the osteons in the Cosserat model. In addition, bone has voids associated with osteocyte lacunae, Haversian canals and others, so that the theory of voids may apply. Experimental evidence thus far is in support of Cosserat elasticity as a generalized continuum representation for fully hydrated bone [5,6] but a contribution of voids to the micromechanics is not excluded.

Recently experiments have been conducted using a square cross section prismatic bar geometry in torsion, using both strain gages [7] and holographic interferometry. These have disclosed substantial differences between wet and dry bone in their micromechanical behavior. Specifically, strain distributions consistent with those predicted by Cosserat elasticity were observed in wet bone, while dry bone behaved nearly classically. Now this torsion geometry is insensitive to the possible presence of microstructural degrees of freedom associated with void mechanics. Effects due to voids are explored in the present experiments.

The present study is directed primarily at the micromechanics of dry bone. The rationale is that dry bone has the same void architecture as wet bone, but the Cosserat-type micromechanical effects appear to be suppressed in dry bone. The micromechanical contributions of the voids may therefore be examined in dry bone.

Experiment. Experiments are based on the predicted size dependence of bending and torsional rigidity of cylindrical rods. In classical elasticity, the rigidity is proportional to the fourth power of the diameter. In Figures 1 and 2, such behavior corresponds to a straight line through the origin. In Cosserat elasticity, size effects are predicted in torsion and bending, in which thin specimens are more rigid than expected classically. The theory of voids predicts such size effects only in bending, as shown in figure 1. It should consequently be possible to distinguish between these theories by measuring the size dependence of bending and torsional rigidity of cylindrical specimens.

The experiments were performed via a special micromechanics apparatus in which the torque was applied to the specimen electromagnetically and the angular displacement measured by laser interferometry. The apparatus permits accurate measurements to be performed upon specimens with a wide range of rigidity, including very thin specimens. In addition, both torsion and bending measurements may be performed in the same apparatus, with identical calibrations. Specimens of both wet and dry human Haversian bone were examined at 23±1 deg C; dry specimens were at ambient humidity, 45±5%. Specimens were cut slowly into a cylindrical shape on a lathe, their rigidity was measured, then they were machined to a smaller diameter and tested again. Specimens were from 3.11mm to 0.66mm in diameter, with the specimen axis aligned with the osteon axis.

Results. Size effects were observed both in torsion and in bending. These effects were substantially smaller in dry bone than in wet bone. Specifically, in wet bone the maximum stiffening effect was about a factor of two in bending and more than a factor of three in torsion. In dry bone, the stiffening effect was from 24% to 34% in bending and from 14% to 33% in torsion. The results are displayed in figure 2.

Discussion and Conclusion. The experiments do not reveal dry bone to behave purely as a continuum with voids. There remain size effects in torsion, which are not predicted by the void theory; these effects are comparable in magnitude to the effects in bending, which are anticipated by void theory. Several hypotheses may be examined in an effort to account for the observations. First, there is the possibility that the length parameter associated with the void theory is too small to give rise to substantial incremental effects in bending in specimens over the size range examined here. This is plausible in view of the fact that the voids in bone are significantly smaller than the osteons, which appear to be responsible for the Cosserat-type effects in wet bone. Second, it is possible that effects due to voids are indeed manifested in the size range studied here, but that the magnitude of the effects is small. There is sufficient freedom within the theory of voids (as well as in Cosserat elasticity) for this to occur, since the new elastic constants in that theory govern not only the size scale associated with novel phenomena, but also the magnitude of such phenomena.

The Cosserat elastic behavior of wet bone has been attributed to the osteons behaving as stiff fibers in a compliant matrix, the cement substance. The suppression of such behavior[7] in dry bone may be ascribed to a stiffening of the cement substance, resulting in the fiber and matrix having similar stiffnesses. The present results suggest that, in view of the continued presence of size effects in torsion, effects due to the osteonal structure have not been completely suppressed in the drying of the bone. It is concluded that dry bone exhibits micromechanical phenomena in addition to those attributable to the presence of voids.

The effect of void mechanics could not be evaluated in isolation from other phenomena in this study. Nevertheless, the possibility remains that such isolation may be performed in experiments involving a size scale yet smaller than that employed here.

References.

[1] Eringen, A.C. "Theory of micropolar elasticity", in Fracture, H. Liebowitz, ed., vol 2, Academic Press, pp. 621-729 (1968).

[2] Cowin, S.C., "Linear elastic materials with voids", J. Elasticity 13: 125-147 (1983).

[3] Cowin, S.C.,"A note on the problem of pure bending for linear elastic materials with voids", J. Elasticity 14: 227-233 (1984).

[4] Lakes, R.S. and Saha, S. "Cement line motion in bone", Science 204: 501-503 (1979).

[5] Yang, J.F.C. and Lakes, R.S. "Experimental study of micropolar and couple stress elasticity in bone in bending", J. Biomechanics 15: 91-98 (1982).

[6] Yang, J.F.C. and Lakes, R.S. "Transient study of couple stress in compact bone: torsion", J. Biomech. Engng. 103: 275-279 (1981).

[7] Park, H.C. and Lakes, R.S., "Strain redistribution with hydration in compact bone: effect of Cosserat mechanics of bone", ASME Biomechanics Symposium Proceedings, Albuquerque, NM, June (1985).

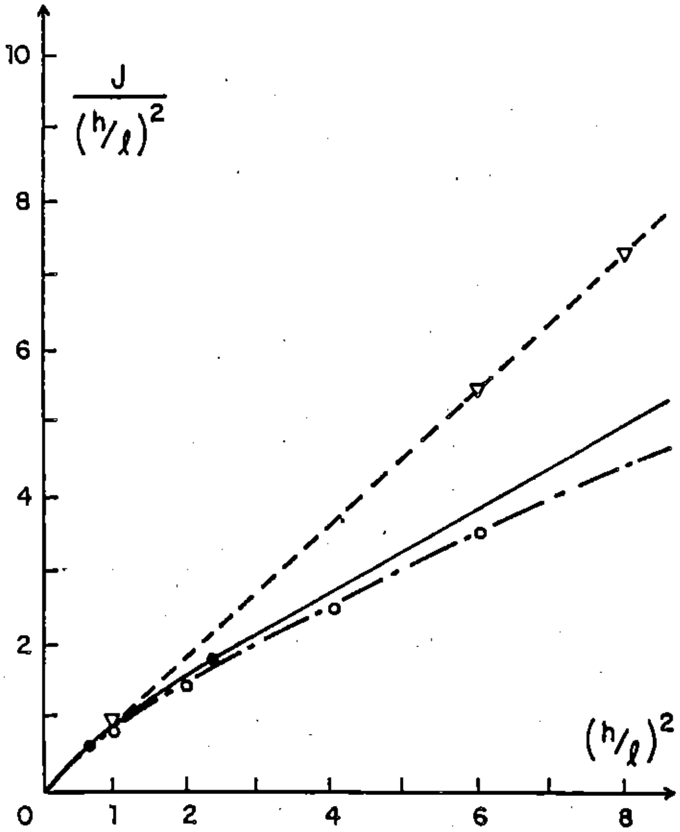

Figure 1. Theoretical predictions of the theory of elastic materials with voids. Bending rigidity of a beam, normalized to the square of the beam depth vs the square of the beam depth h, normalized to the length parameter l. A classically elastic material would exhibit a straight line plot through the origin. The torsional rigidity of a material with voids is classical.

Triangles, H=0.5, R=0.7, maximum stiffening, a factor 1.11.
Solid circles, H=0.95, R=0.8, maximum stiffening, a factor 1.9.
Open circles, H=0.95, R=0.95, maximum stiffening, a factor 2.25.
The quantities H and R are defined in [2], [3].

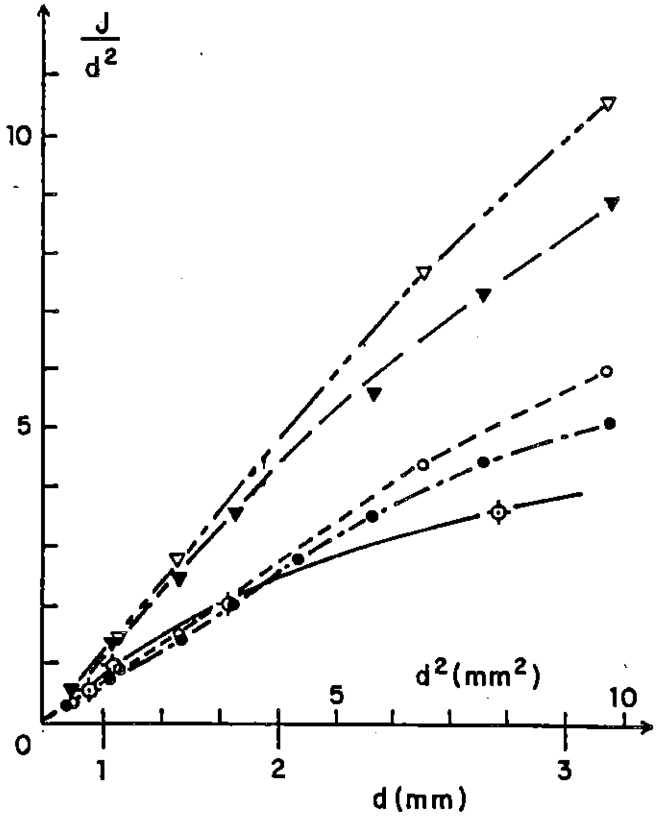

Figure 2. Experimental results for dry and wet bone. Rigidity divided by the square of the diameter vs the square of the diameter.

Triangles, pure bending, dry bone.
Circles, torsion, dry bone.
Circles with crosses, torsion, wet bone.