EXPERIMENTAL MICRO MECHANICS METHODS FOR CONVENTIONAL AND NEGATIVE POISSON'S RATIO CELLULAR SOLIDS AS COSSERAT CONTINUA

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Abstract

Continuum representations of micromechanical phenomena in structured materials are described, with emphasis on cellular solids. These phenomena are interpreted in light of Cosserat elasticity, a generalized continuum theory which admits degrees of freedom not present in classical elasticity. These are the rotation of points in the material, and a couple per unit area or couple stress. Experimental work in this area is reviewed, and other interpretation schemes are discussed. The applicability of Cosserat elasticity to cellular solids and fibrous composite materials is considered as is the application of related generalized continuum theories. New experimental results are presented for foam materials with negative Poisson's ratios.

1. Continuum theories: degrees of freedom

The question of how much freedom is to be incorporated in an elasticity theory must ultimately be decided by experiment. However, during the development of the theory of elasticity, it was by no means obvious how much freedom was necessary to describe materials. The development was guided by physical models of material deformation. For example, the early uniconstant theory of Navier is based upon the assumption that forces act along the lines joining pairs of atoms and are proportional to changes in distance between them [1]. This theory entails a Poisson's ratio of 1/4, so that there is only one independent elastic constant for an isotropic solid. The constitutive equation is given in Eq. 1. Symbols are defined below.

$$\sigma_{kl} = G\,\varepsilon_{rr}\,\delta_{kl} + 2G\,\varepsilon_{kl} \qquad (1)$$

This uniconstant theory was used by Navier, Cauchy, Poisson, and Lamé during the early days of the theory of elasticity. The uniconstant theory was rejected based on experimental measurements of Poisson's ratio, which is close to 1/3 in most common materials. The issue was not decided immediately, since decisive experiments were difficult to perform in the late 1800's.

We now use the following constitutive equation for classical isotropic elasticity [2,3], in which there are the two independent elastic constants λ and G.

$$\sigma_{kl} = \lambda\varepsilon_{rr}\,\delta_{kl} + 2G\,\varepsilon_{kl} \qquad (2)$$

The Poisson's ratio $\nu = \lambda/2(\lambda + G)$ is restricted by energy considerations to have values in the range from -1 to 1/2. Ordinary materials have a positive Poisson's ratio, so that for many years the range from zero to 1/2 was considered the physically acceptable range [3]. Recently, a new class of cellular solids with a negative Poisson's ratio has been developed [4,5], extending the range for ν to -0.7 and below.

More freedom is incorporated in the Cosserat theory of elasticity, also known also as micropolar elasticity [6,7,8,9]. This theory incorporates a local rotation of points as well as the translation assumed in classical elasticity; and a couple stress (a torque per unit area) as well as the force stress (force per unit area). The force stress is referred to simply as 'stress' in classical elasticity in which there is no other kind of stress. The idea of a couple stress can be traced to Voigt in the late 1800's during the formative period of the theory of elasticity. In the isotropic Cosserat solid there are six elastic constants, in contrast to the classical elastic solid in which there are two, and the uniconstant material in which there is one. Certain combinations of Cosserat elastic constants have dimensions of length and are referred to as characteristic lengths. The constitutive equations for a linear isotropic Cosserat elastic solid are :

$$\sigma_{kl} = \lambda\varepsilon_{rr}\,\delta_{kl} + (2\mu + \kappa)\varepsilon_{kl} + \kappa\varepsilon_{klm}(r_m - \varphi_m) \qquad (3.1)$$

$$m_{kl} = \alpha\,\varphi_{r,r}\,\delta_{kl} + \beta\varphi_{k,l} + \gamma\varphi_{l,k} \qquad (3.2)$$

The usual summation convention for repeated indices is used throughout. σ_{kl} is the force stress, which is a symmetric tensor in Eqs 1 and 2 but it is asymmetric in Eq. 3. m_{kl} is the couple stress, $\varepsilon_{kl} = (u_{k,l} + u_{l,k})/2$ is the small strain, u_k is the displacement, and e_{klm} is the permutation symbol. The microrotation φ_k in Cosserat elasticity is kinematically distinct from the macrorotation $r_k = (e_{klm}u_{m,l})/2$. In three dimensions, the isotropic Cosserat elastic solid requires six elastic constants λ, μ, α, β, γ, and κ for its description. The following technical constants derived from them are more beneficial in terms of physical insight. These are [7,8]: Young's modulus $E = (2\mu+\kappa)(3\lambda+2\mu+\kappa)/(2\lambda+2\mu+\kappa)$, shear modulus $G = (2\mu+\kappa)/2$, Poisson's ratio $\nu = \lambda/(2\lambda+2\mu+\kappa)$, characteristic length for torsion $l_t = [(\beta+\gamma)/(2\mu+\kappa)]^{1/2}$, characteristic length for bending $l_b = [\gamma/2(2\mu+\kappa)]^{1/2}$, coupling number $N = [\kappa/2(\mu+\kappa)]^{1/2}$, and polar ratio $\Psi = (\beta+\gamma)/(\alpha+\beta+\gamma)$. The range in Poisson's ratio is from -1 to +0.5, the same as in the classical case [10]. When α, β, γ, κ vanish, the solid becomes classically elastic. The case N = 1 (its upper bound) is known as 'couple stress theory' [8,11].

Other generalized continuum theories have been developed, however we do not explore them in much detail here since very few experimental studies have been done with them. Such theories include the following. Microstructure [12] or micromorphic [13] elasticity permits the points in the solid to deform microscopically as well as to translate and rotate, leading to 18 elastic constants in the isotropic case. Cosserat elasticity is a special case of microstucture elasticity. A different special case is a theory for elastic materials with voids, which makes use of the dilatation of points rather than their rotation as an extra kinematical variable [14]. The complexity of void theory is comparable to that of Cosserat elasticity. Nonlocal elasticity permits the stress at a point to depend on the strain in a region around that point [15]. In nonlocal theory the mechanical property is a function of position rather than a number; for particular forms of this function, a characteristic length can be defined in terms of the effective range.

Constitutive equations 1 - 3 are equally consistent mathematically, so the only way to determine which is the most appropriate is by experiment. In the early development of the theory of elasticity, experiments were directed toward discriminating Eq. 1 from Eq. 2. This was done by measuring the Poisson's ratio in isotropic solids. The analytical basis for discriminating Eq. 2 from Eq. 3 was developed beginning in the 1960's while the experimental work is relatively recent. It is the purpose of this article to review and unify the experimental work in this area, and to point to new directions with novel classes of materials.

1.1 Behavior of Cosserat elastic solids

Salient consequences of Cosserat-type theories are as follows:

(i) A size-effect is predicted in the torsion of circular cylinders of Cosserat elastic materials. The effective shear modulus associated with such cylinders increases as their size decreases [10]. A similar size effect is also predicted in the bending of plates [10] and of beams [16]. By contrast, a material obeying the continuum theory of voids [14] is predicted to exhibit size effects in bending but not in torsion. No size effect is predicted in tension [10].

(ii) Calculation of stress concentration factors around a circular hole, taking into account couple-stresses, results in lower values than accepted heretofore [17]. Stress concentration around a rigid inclusion in an elastic medium is greater in a Cosserat solid than in a classical solid. The maximum stress occurs at the interface rather than in the medium itself, in a Cosserat solid [18]. Stress concentration near cracks and elliptic holes is reduced in comparison to classical predictions [19-23].

(iii) Dilatational waves propagate non-dispersively, i.e. with velocity independent of frequency, in an isotropic Cosserat elastic medium. Shear waves propagate dispersively in the presence of couple-stress [7]. A new kind of wave associated with the micro- rotation is predicted to occur in Cosserat solids [7]. Dispersion of dilatational waves can be accounted for in the more general Cosserat-type theories known as microstructure elasticity [12] or micromorphic elasticity [13].
(iv) The mode structure of vibrating Cosserat bodies is modified from that of classical elastic bodies[11].

2. Physical origin of mechanical behavior

Both Cosserat-type theories and classical elasticity are continuum theories which make no reference to atoms or other structural features of the material which is described. Nevertheless, consideration of structural information for a particular material can lead to a greater understanding of its behavior and to quantitative prediction of its mechanical properties. The physical origin of stress lies in the interatomic forces of attraction and repulsion. Elasticity theory represents more than an analytical description of the phenomenological behavior since it can be derived as a first approximation of the interaction between atoms in a solid [24]. Interatomic forces are short range; but they exert influence farther than one atomic spacing. Therefore, there must be some resistance to lattice curvature in all solids [24]. The characteristic length l in this case should be on the order of the atomic spacing. Such a characteristic length would be imperceptible in any macroscopic mechanical experiment, but may have relevance in elastic crack tip problems but not in elastic-plastic ones.

Phenomena associated with Cosserat elasticity are likely to be of larger magnitude, and therefore of greater interest in materials such as composites and cellular solids with larger scale structural features. In fibrous composites, the characteristic length l may be the on the order of the spacing between fibers [25]; in cellular solids it may be comparable to the average cell size. [26]. The physical origin of such effects is in the bending and twisting moments transmitted through the fibers in a composite or in the cell ribs in a foam. A spatial average of these moments corresponds to the couple stress m_{kl} in Eq. 3.2. A schematic diagram of force increments upon ribs (in the structural view) corresponding to stress (in the continuum view) and moment increments corresponding to couple stress is shown in Fig. 1. Size effects could also arise as a result of surface tension, however we expect that surface tension will have a significant effect only for very soft, or semi-liquid materials. Structure, however, does not necessarily lead to Cosserat elastic effects. Composite materials containing elliptic or spherical inclusions are predicted to have a characteristic length of zero [27,28].

It is important to distinguish the continuum view from the structural view. The continuum view is useful for making engineering predictions and for visualizing global response of materials, while the structural view is relevant to the underlying causes of the behavior. A connection between these views is achieved by creating an analytical model of the material microstructure, and developing approximations such as series expansions for local deformation fields in order to obtain average values. When only the lowest order terms are retained in such analysis, classical elasticity is obtained as a continuum representation. When higher order terms are retained, a generalized continuum representation (such as Cosserat elasticity) is obtained. In either case the predicted elastic constants are functions of the structure and properties of the constituents; this is how the microphysics is introduced.

3. Experimental Procedures for Cosserat solids

3.1 Methods based on size effects

In a classically elastic rod, the bending and torsion rigidity are proportional to the fourth power of the diameter. In a thin plate the bending rigidity is proportional to the third power of the thickness. By contrast the rigidity depends on size in a different way in Cosserat elastic materials: thin specimens are more rigid than would be expected classically. It is possible to determine one or more of the Cosserat elasticity constants by measurements of specimen rigidity vs size. This approach, which we call the method of size effects, has been used in many experimental efforts both old and recent. Analytical solutions predicting size effects in various geometries are summarized in Table 1. These solutions do not address surface tension as a separate phenomenon. The Cosserat elastic constants which can be extracted are also shown.

TABLE 1

Configurations for determining elastic constants of a Cosserat solid

Configuration	Elastic constants	Ref
Tension. No size effects if isotropic	E, ν	[10]
Bending of plate, cylindrical bend	E, l_b	[10]
Bend circular plate, clamped edge	E, l_b ,N	[29]
Bend curved bar	E, l_b ,N	[30]
Bend rod: vary diameter	E, l_b ,N	[16]
Torsion of rod: vary diameter	G, l_t ,N, Ψ	[10]
Torsion of square bar	G, l_t ,N	[31]

In the method of size effects, any instrumentation capable of determining bending and/or torsion rigidity is appropriate. Since rigidity is to be determined over a considerable range, it is necessary to make sure that parasitic errors such as those due to instrument friction are minimized or eliminated. For example, load may be applied electromagnetically or by dead weights. In the case of cantilever bending, there would be no problem with dead weight loading; however for torsion, the pulleys used to redirect the load could introduce errors due to friction. Such errors would be more important for thin specimens and would obscure the size effects. Deformation measurement by holographic interferometry, other optical methods, strain gages, and free core LVDT's are all suitable since there is no friction error. Use of spring loaded LVDT's with sleeve bearings would by contrast introduce errors more prominent in thin specimens.

Most authors have used these methods in such a way as to obtain only one Cosserat elastic constant. In that regard, a particular form of the method of size effects was found to be useful by the present author. In this approach the rigidity of the same specimen was tested in both torsion and pure bending using the same apparatus [32], which makes use of electromagnetic torque generation and interferometric detection of angular displacement. Each specimen was then cut to a smaller size and the rigidities again determined. This approach is capable of determining all six of the Cosserat elastic constants. Moreover, cross verification of the results is possible, in a manner similar to the measurement of E, G, and ν in classical elasticity, with verification of their interrelation.

3.2 Specimen preparation for cellular solids

The method of size effects makes use of a set of specimens of different diameter or thickness. If the characteristic lengths are small, it is desirable to examine thin specimens. The preparation of these may require special care. Stiff porous materials, such as bone and the stiffer polymer foams, may be successfully cut on a lathe using conventional cutting tools. Specimens thinner than about 3 mm down to 0.2 - 0.5 mm are prepared on a lathe by an abrasive machining technique in which the lathe is operated at high speed and a strip of emery cloth is applied by a small force to the surface. Flexible polymer foams can be cut into circular cylinders by use of a coring tool driven by a power drill. The coring tool consists of a thin wall metal tube with a sharpened end. The re-entrant specimens are of square section as prepared; smaller rectangular section specimens can be cut from these by compressing the foam between platens and cutting with a scalpel.

3.3 Data reduction in the method of size effects

The method of extraction of the Cosserat elastic constants from the experimental size-effect data is as follows. The method makes use of the exact analytical solutions for the geometry used in the experiments, i.e. torsion and pure bending of a circular cylindrical rod of a Cosserat elastic material. For torsion[10], the ratio of rigidity to its classical value is

$$\Omega = 1 + 6(l_t/a)^2\ [1 - 4\Psi X/3)/(1 - \Psi X)], \qquad (4)$$

with a as the rod radius, $\Psi = (\beta + \gamma)/(\alpha + \beta + \gamma)$ and $X = I_1(pa)/I_0(pa)$, and $p^2 = 2\kappa/(\alpha + \beta + \gamma)$. I_1 and I_0 are the modified Bessel functions of the first kind. A special case of interest is for N = 1 ($\kappa \rightarrow \infty$) in which the [] bracket in equation 4 becomes unity; this is for 'couple stress elasticity'.

For bending [16] of a rod of radius a, the rigidity ratio Ω is

$$\Omega = 1 + 4(l_b/a)^2\ (1 - \beta/\gamma)^2 + \left[(8N^2(\beta/\gamma + \nu)^2 / (\zeta(\delta a) + N^2(1 - \nu))(1 + \nu)\right] \qquad (5)$$

with $\zeta(\delta a) = (\delta a)^2\ [(\delta a\, I_0(\delta a) - I_1(\delta a)) / (\delta a\, I_0(\delta a) - 2I_1(\delta a))]$, and $\delta = N/l_b$.

Plots of rigidity divided by the square of the diameter vs. the square of the diameter for torsion (figure 2) and bending (figure 3) are used. It is to be emphasized that these are not stress strain curves. In classical elasticity, the plot is a straight line through the origin; the slope is proportional to the shear modulus G in the torsion case and the Young's modulus E in the bending case. Comparison of experimental plots and theoretical curves permit the determination of E and G in the same manner as is done in classical elasticity. The offset between the experimental curve and the asymptotically parallel theoretical curve yields the "characteristic length" from the torsion plot (Figure 2). Similarly, a second characteristic length is obtained from the bending plot (Figure 3). The offset used to determine the characteristic length can be found graphically or by numerical procedures. The shape of the torsion plot is then used to extract the coupling number N, as shown in Figure 2. A large value of N (the upper bound is 1) leads to a large apparent stiffening for slender specimens. The structure of the torsion plot in the vicinity of the origin is used to determine Ψ. In practice, the present investigator has used a numerical algorithm to minimize the mean-square deviation between the experimental data and the theoretical graphs, to extract the elastic constants.

3.4 Wave and field methods

Micromechanical effects describable by Cosserat elasticity occur in the propagation of elastic waves. The theory suggests the Cosserat elastic constants can be extracted from measurements of wave dispersion [7]. Such a procedure is warrantable in the case of elastic materials with no attenuation of waves. It is more difficult to apply such methods to composite materials and cellular solids since both viscoelastic (time-dependent) and Cosserat (spatial or gradient sensitive) effects will contribute to the dispersion of waves. Viscoelastic behavior, therefore constitutes a confounding variable in wave methods, unless the material studied has low loss, or a correction is made for dispersion of viscoelastic origin. By contrast, in the method of size effects, viscoelasticity can be effectively decoupled by using isochronal (constant time) or constant frequency data. However, some of the constants in the micromorphic theory, as well as the microinertia [7] in Cosserat elasticity, can only be determined dynamically.

Micromechanical effects also occur in the distribution of stress and strain under various conditions, leading to the concept of field methods as contrasted to the global rigidity methods described above. A method of this type involves measuring, in the linear range of behavior, the strain distribution or displacement field around a stress raiser such as a circular hole, and comparing the results with theoretical distributions from classical elasticity [2,3] and from Cosserat elasticity [7,8,17]. Use of stress concentration factors for fracture, while of great practical importance, is of limited use in evaluating Cosserat elastic solids, since phenomena such as plastic and damage zone formations also influence the results. Cosserat type damage theories, however, may be examined in this vein. Another field type method is based on the predicted warp and strain distributions on the surface of a square cross-section prismatic bar in torsion. In a Cosserat solid, nonzero stresses and strains are predicted at the corners of the cross section, in contrast to the results of classical elasticity [31]. Moreover, the warp of cross sections is predicted to be reduced in Cosserat solids [31]. A further field approach involves the determination of the displacement field near a concentrated load upon the surface of a large block, modeled as a half space in the 'Boussinesq' problem [3]. This has some practical significance in terms of determining the indentability of cellular materials including honeycombs, which are vulnerable to concentrated loads. In all these approaches, deviations from classical elasticity are greatest if both N and the characteristic lengths are large.

If l is small, problems arise both in field methods and in methods based on size effects. In the method of size effects, preparation of very thin specimens can present difficulties. In field methods the specimen itself can be large, but it is difficult to conduct accurate measurements of strain or displacement fields in the presence of large strain gradients. These very gradients, however, reveal the Cosserat elastic effects sought. All known experimental methods involve averaging over some nonzero length, eg. the gage length in the case of strain gages. Nevertheless, study of the strain (or displacement of a small crack) of the corner element in torsion represents a useful null experiment which can be used for screening.

4. Cosserat viscoelasticity

In viscoelastic materials, the mechanical property values may be regarded as complex quantities (with magnitude and phase) which depend on frequency, in constitutive eq's 1-3. The mechanical properties can also be regarded as functions of time, in which case the constitutive equations assume a convolution integral form. Both the size effect and field type experiments can be performed at different frequencies, or in a transient form in which the response to step load or step deformation is evaluated as a function of time. In these methods, it is straightforward to separate the effects of time/frequency and of Cosserat elasticity. For example, separate size effect plots can be generated at different times or frequencies. By contrast in the wave approach, such separation is not possible since wave frequency is the only free variable accessible to the experimenter; the wavelength is then determined by the properties of the medium.

5. Experimental Results

5.1 Review of materials as Cosserat solids

Results for the mechanical properties of a variety of materials as obtained by several authors are as follows, in Table 2. All experiments were done quasistatically unless otherwise stated.

Table 2
Classical and Cosserat elastic technical constants of materials

	Elastic constants									
	Classical		Cosserat					Structure		
Material	E(MPa)	G(MPa)	ν	l_t(mm)	l_b(mm)	N^2	Ψ	Ref	Size	Comment
Aluminum	73000	--	--	--	<0.03	*	--	[33]		Bend plate
Aluminum	69000	--	--	--	<0.05	*	--	[34]	0.05mm	Bend plate
Steel	212000	--	--	--	<0.05	*	--	[34]	0.05mm	Bend plate
KNO_3	36000	--	--	$6x10^{-8}$	0.03		[35]	atomic	Waves	
Foam, PVC	--	2.8	--	0.95	--	*	--	[36]	1mm	Resonance shear thickness
Epoxy/ aluminum particle	--	7000	--	0	--	--	--	[10]	1.4mm	Torsion, rod Classical
PMMA	--	1000	--	0	--	*	--	[37]	0.1nm	Torsion, rod
Human bone	12000	4000	--	0.22	0.45	≥0.5	1.5	[37-39]	0.2mm	Bend, torsion Anisotropic
Graphite, H237	4500	--	0.06	1.6	2.8	*	--	[40]	1.6mm	Bend bar
Foam, PS	0.6	1.1	0.07	3.8	5.0	0.09	1.5	[41]	1mm	Bend, torsion of rods
Foam, dense polyurethane	300	104	0.4	0.62	0.33	0.04	1.5	[42]	0.18mm	Bend, torsion of rods
Foam, syntactic	2758	1033	0.34	0.065	0.032	0.1	1.5	[42]	0.15mm classical	Bend, torsion Nearly
Re-entrant foam, poly-urethane	0.02	--	-0.7	--	≈2	--	--		≈0.8mm	Bend bar

*: Interpretation based on couple stress theory for which N = 1 (its upper bound) by assumption.

It is of interest to consider the structure of these materials. The steel, aluminum, and graphite are polycrystalline, but the nature of the grain boundaries is likely to be different in the graphite. The potassium nitrate is a single crystal and the relevant structure is on the atomic scale as was the experimental method in this case. The results for potassium nitrate were obtained by interpretation of wave propagation data from solid state physics. The polymethyl methacrylate (PMMA) is amorphous, and the relevant structure is on the atomic/ molecular scale. It was used as a control in mechanical size effect experiments upon bone. Human bone is the most complex of the materials. It has particulate structure on the scale of less than one µm, microfibrous structure, lamellar structure, pores from 0.3 µm to 100µm in size, and large fibers known as osteons, which are 150 µm to 250 µm in diameter. Both the dense polyurethane and syntactic foams are dense closed cell foams. The syntactic foam consists of hollow glass microballoons in an epoxy matrix and therefore has much in common (except for density) with particulate composites. The polystyrene (PS) foam is a low density closed cell foam. Re-entrant foam contains cell ribs which protrude inward and are convoluted in comparison with those of conventional foams.

Structure (which entails heterogeneity) is without doubt the cause responsible for microelastic phenomena. Not all kinds of structure, however, lead to such phenomena. We may attribute the failure of several early experiments to reveal Cosserat elasticity to the structure of the materials chosen. Particulate solids with spherical or ellipsoidal inclusions are predicted to have characteristic lengths of zero [27,28].

As for experimental procedures and interpretation, we remark that the method of size effects was used for most of these studies. In the case of the PVC foam, a dynamic thickness-shear vibration technique was used; a potential confounding variable is the viscoelasticity of these materials, which results in a change of stiffness with frequency. This is important since the frequency varied as the thickness was changed. The static size effect results did not suffer this limitation. In the graphite, bend tests were done on square cross section specimens of different size. No torsion tests were done but the ratio β/γ (in the symbols used in Equation 3) was inferred from the shape of the bend size effect curve. These results were complicated by the material nonlinearity of graphite. Linearity of response was verified in all of the size effect studies (at small strain) performed by the present author, as well as by several others. Error analysis was performed in only a few of these experimental articles. For example, in bone [37], most specimens were Cosserat elastic within a confidence interval of better than 99%. In dense polyurethane foam [42], the residual (mean square deviation between theory and experiment) was a factor 172 better for a Cosserat interpretation of torsion than for a classical interpretation; and a factor 10 better for Cosserat bending.

As for interpretation, several authors used 'couple stress theory' for interpretation. This is a special case of Cosserat elasticity corresponding to N = 1. Results of this type were converted into the Cosserat form in Table 2. The characteristic lengths are defined somewhat differently in these theories, so that the Cosserat length for torsion is $\sqrt{3}$ times the couple stress length. Stress analysis is not significantly easier in couple stress theory than in Cosserat elasticity [9].

6. Verification of Cosserat elastic constants

The objective of stress analysis, regardless of the constitutive equation assumed, is to quantitatively predict stress and deformation fields for objects with shapes different from that used in the experiment to determine the elastic constants. In this vein, it is of interest to examine the predictive power of Cosserat elasticity and other generalized continuum theories. For example, Cosserat elastic constants derived from size effects in human bone were used to predict surface strain distributions around holes in a strip under tension and on prismatic bars under torsion. For the holes, reasonable qualitative agreement was found, but it was not perfect owing to the neglect of the anisotropy of bone [43]. Good quantitative agreement was obtained for strain distributions in square cross section bars of bone in torsion [44] since the same elastic constants l_t and N are relevant in this geometry as in the torsion size effect study. Bone is anisotropic, with a symmetry which is hexagonal or transversely isotropic. Consequently, with the specimen aligned the same way in both cases, the same elastic constants appear in both cases.

Another interesting effect in the torsion of square cross section bars is in the displacement of a small notch in the corner of the cross section. The displacement should be zero in the classical case, since the stress is zero at the corner; but it should be nonzero in the Cosserat case, since the corner stress is nonzero [31]. Experimentally in holographic studies, such corner notch displacement was zero in solid polymethyl methacrylate (PMMA), but was nonzero in dense polyurethane foam [45] which was shown by size effect studies to be Cosserat elastic [32]. Similar displacements were easily observed visually in large cell foams [46] which had also been previously identified as Cosserat elastic [41].

6.1 Cosserat viscoelasticity

In viscoelastic materials, any of the material properties can depend on time or frequency. In Cosserat solids, this includes the characteristic lengths. In human bone, the torsional characteristic length was found to be a factor of 1.6 larger under quasistatic conditions equivalent to 0.1 Hz [37] than at 32 kHz [47]. This phenomenon is attributed to the viscoelastic attributes of the cement substance between the large fibers (osteons) in bone.

6.2 Study of materials as other generalized continua

The above results are also relevant to the generalized continuum theory of elastic materials with voids [14]. The theory, intended as a representation of porous solids, makes use of a micro volume change rather than a micro rotation as an extra kinematical variable. The theory predicts size effects to occur in bending but not in torsion. Those experimental results in Table 2 which disclosed size effects in torsion [l_t >0] imply that the continuum theory of voids is not an appropriate description. Materials in that category include all of the cellular solids in Table 2.

A few experimental studies have addressed the question of whether other possible generalized continuum models are applicable. As for nonlocal theory, measurements of dispersion (dependence of velocity on frequency) of ultrasonic waves have been interpreted with the aid of nonlocal elasticity theory [48,49]. The dispersion involves a reduction in wave speed with an increase in frequency, which is the opposite of the dispersion seen in viscoelastic materials. A nonlocal characteristic length of about 0.3 mm was inferred for particleboard, and this value was correlated with the fracture toughness.

6.3 Materials with negative Poisson's ratios

A new class of cellular materials was recently developed by the present author; these materials exhibit a negative Poisson's ratio [4,5]. The materials are prepared from conventional low density foams by achieving a triaxial permanent compression which causes the cell ribs to protrude inward [4]. In thermoplastic polymer foams the triaxial compression is followed by heating beyond the softening point. In ductile metal foams, sequential plastic deformations in three directions are applied. The expanded range of Poisson's ratios represents more freedom than is usually expected in typical elastic materials for which the Poisson's ratio is not very different from 1/3. This freedom is, nevertheless, within the framework of classical elasticity. In both classical [2] and Cosserat [7] solids, the range of the Poisson's ratios allowed for an isotropic material based on energy considerations is from -1 to 1/2. A coarse cell structure is not required to achieve a negative Poisson's ratio [50]. The Poisson's ratio is a classical concept, and there is no length scale in classical elasticity. However the cell size is relevant to microelastic phenomena. In particular, we hypothesize effects (of the type discussed above in section 1.1) describable by Cosserat elasticity and microstructure elasticity in these materials. The rationale for this hypothesis is that the cell ribs in these materials are convoluted. Consequently the bending and twisting moments in them are likely to be more important than normal and shear forces, in comparison with conventional foams which have relatively straight ribs. Moreover, the convoluted ribs are expected to have a lower resonant frequency than that of straight ribs.

The following experiments were conducted: standing wave studies to explore microstructure elasticity effects, and quasistatic experiments to examine Cosserat elasticity. The material examined was Scott industrial foam, transformed to a re-entrant structure. The Scott foam is a polyurethane open cell reticulated foam. In the wave studies, micro-vibration of the ribs manifested itself in the form of wave dispersion and cut off frequencies of wave motion [51]. The phenomena are associated with microstructure elasticity, however only one of the 18 elastic constants (in addition to a stiffness) can be extracted from such data. In a static experiment, the warp of square cross section bars of foam was examined. The torsional deformation of a bar of negative Poisson's ratio foam is shown in Fig. 4. The warp is considerably reduced in comparison with that of a classical bar (not shown), and the reduction is greater for the material with larger cells(hence a larger characteristic length). Such reduction of warp is predicted to occur in a Cosserat solid [31]; the effect has nothing to do with the Poisson's ratio, since the only classical elastic constant involved in the torsion problem is the shear modulus. Preliminary bending studies were also performed using dead weights for load application and a measuring microscope to determine the displacement. The bending characteristic length for re-entrant foam with 0.8 mm call size was about 2 mm as shown in Table 2.

7. Empirical models for composite materials

This article deals with different constitutive representations of linearly elastic behavior. Neither nonlinearity nor fracture are included. However, with the aim of some degree of completeness in connection with nonclassical effects, we discuss here different approaches for dealing with such effects in the context of fracture.

A large body of empirical data dealing with fracture of composite materials with stress concentrators [52,53] reveals a common theme. Observed stress concentration factors and stress intensity factors are consistently lower than values predicted classically, even with proper accounting for anisotropy. Large holes (25 mm in diameter) exhibit stress concentration factors in reasonable agreement with classical theory [54]; smaller holes exhibit smaller values [52-54] and very small holes cause virtually no reduction in strength [54].

A variety of fracture models have been proposed to describe the observed phenomena. One criterion for fracture assumes that fracture occurs if the <u>average</u> stress over some distance a_0 becomes equal to the ultimate strength of the material [52,53,55]. In a second criterion, fracture occurs when the stress some distance d_0 from the stress raiser becomes equal to the ultimate stress. This is the so-called <u>point stress</u> criterion. It is tacitly assumed that these 'characteristic distances' are material parameters and are thus independent of the size of the discontinuity. The various empirical models are generally found to agree with the experimental results, for particular configurations. This is not surprising since the procedure is basically one of curve fitting. However, fracture tests on quasi-isotropic graphite-epoxy laminates of different stacking sequences, with holes of different size disclosed that the strength depends on the stacking sequence [56]. Moreover, d_0 was found not to be a material constant but is related to the square root of the hole radius [56].

Studies of strain fields around holes and notches in composites loaded below the yield point have disclosed results similar to the above fracture studies. Large holes exhibit strain fields in good agreement with classical theory [57], while strain around smaller holes and notches is smaller than expected [43,55]. For small elliptic holes in graphite epoxy composite the discrepancy was a factor of six [58].

8. Discussion

Experimental approaches to structured materials as generalized continua, particularly Cosserat solids, have been reviewed. Many common materials including steel, aluminum and particulate composites behave essentially classically, that is, with characteristic lengths of zero. Cellular solids of various type obey Cosserat elasticity, but their behavior is not consistent with the continuum theory of voids. As for structure-property relations, there remain unanswered questions concerning open <u>vs</u> closed cells, and the effect of a re-entrant transformation on the micromechanics of cellular solids. Structural theories predict characteristic lengths less than the cell size in most cases, but experimental results indicate otherwise for several materials. An intriguing question is whether the characteristic length can be made much larger than the structure size.

Phenomena in qualitative agreement with Cosserat elasticity have been reported in composite materials, however these have been modeled, in the fracture case, by empirical models. The empirical models are <u>ad hoc</u>. When additional experimental variables are introduced, e.g. specimen width, further adjustable parameters must be introduced. Moreover, the fracture models do not address the issue of strain distributions around inhomogeneities and stress raisers. We therefore perceive a need for a method for the rational prediction of strain and stress fields in composites, as well as failure processes. The approach to be taken should incorporate in some way the actual micro-deformation processes in the composite. The generalized continuum theories described above contain the necessary freedom.

Investigation of the generalized continuum aspects of structure-property relations has in part inspired the development of a new class of materials with negative Poisson's ratios. In connection with these developments, we concur with the suggestion that a closer collaboration between the materials sciences and the solid mechanics disciplines is warranted [59]. The negative Poisson's ratio materials exhibit Cosserat type effects as well as dynamical microelastic effects. One of the most important predictions of Cosserat elasticity is an increased toughness, and a relative immunity to the effects of stress concentrations. Enhancedtoughness can also come about as a result of the negative Poisson's ratio. Future studies will therefore deal with enhancement of Cosserat effects in materials such as re-entrant foams, and improvement in their toughness.

9. Conclusions

1 The method of size effects is of use in elucidating nonclassical elastic behavior, and in determining Cosserat elastic constants.

2 In the present survey of experimental work, it is observed that polycrystalline and particulate type material microstructures behave classically or nearly classically.

3 Cellular structures in materials, by contrast, give rise to microelastic effects for which Cosserat elasticity is a reasonable description.

4 The continuum theory of voids is not successful in describing the microelastic effects observed in cellular solids.

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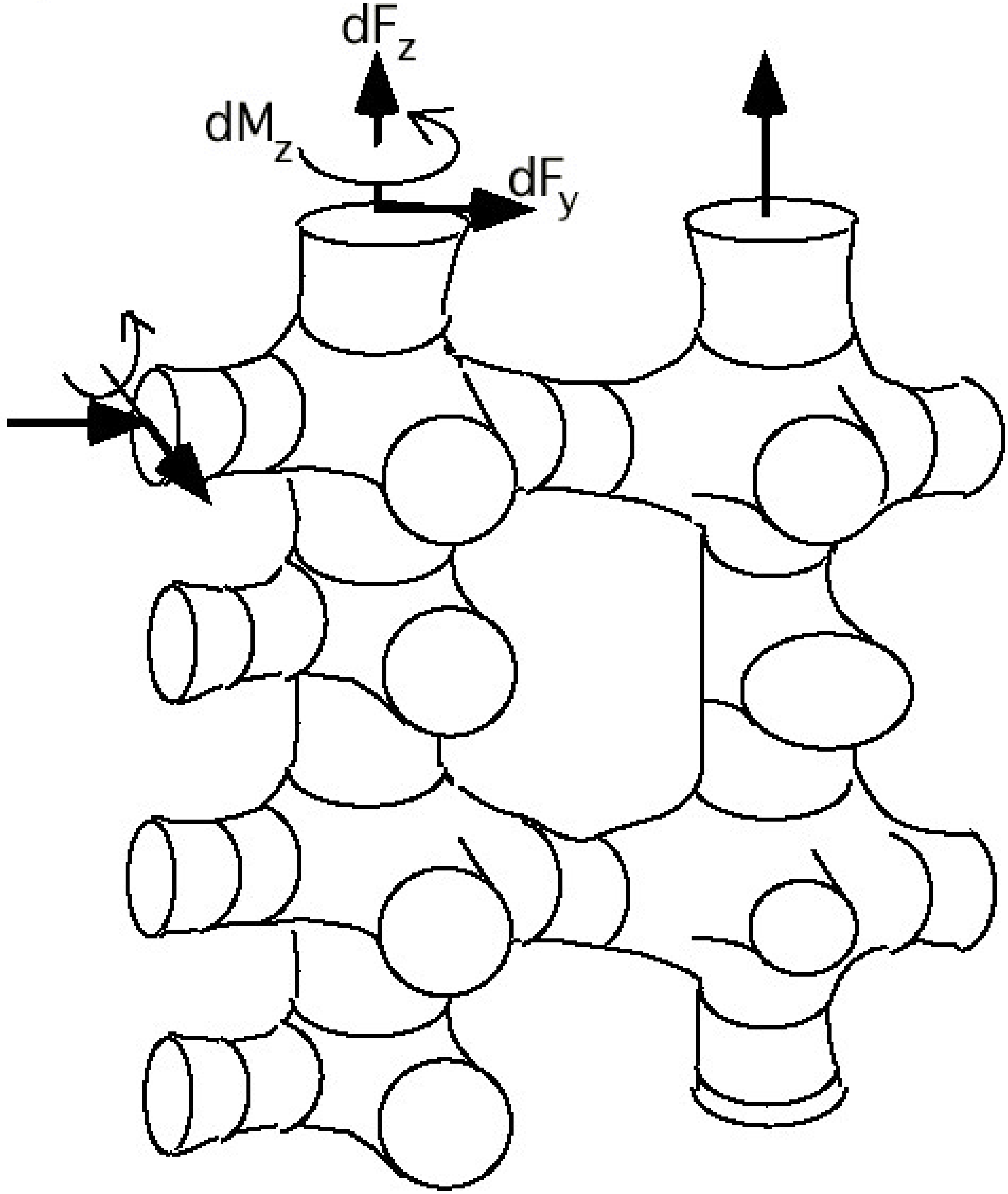
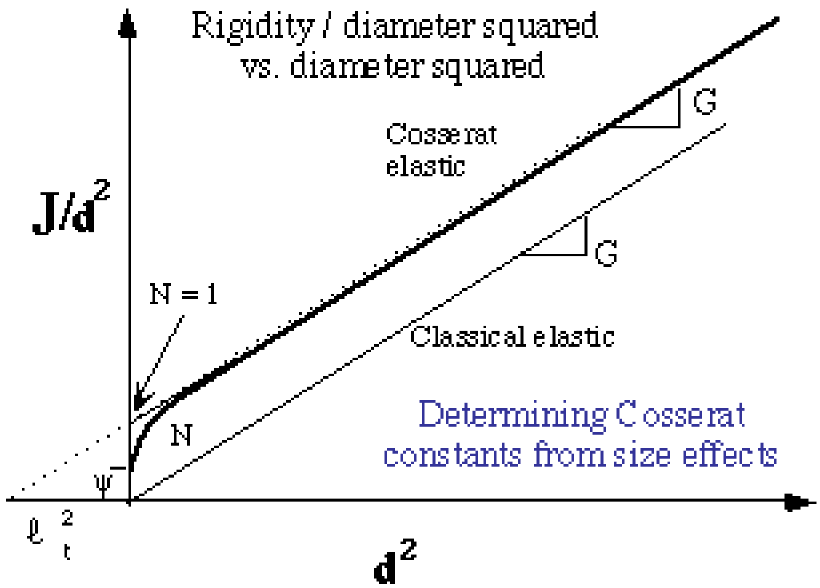
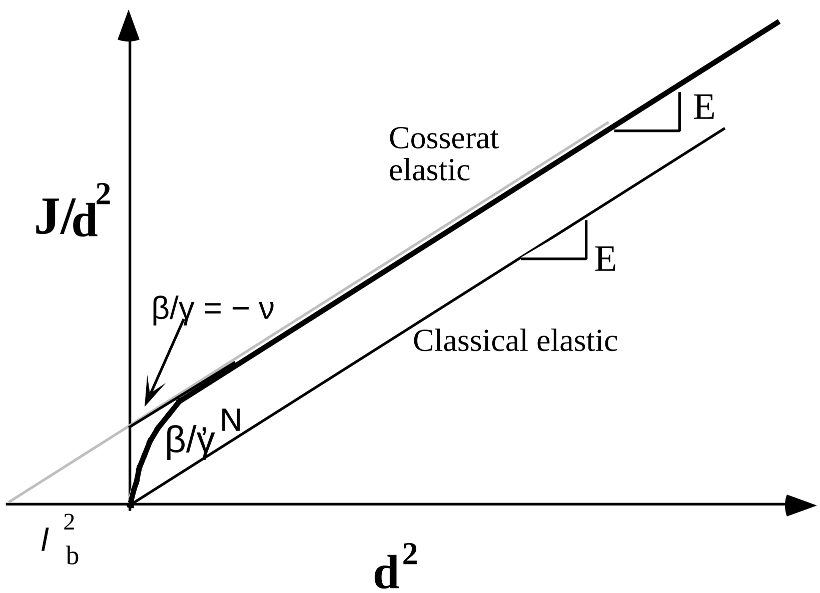
List of figures





2 Extraction of elastic constants from size effect data in torsion of a circular cylindrical rod. Rigidity/ diameter squared vs diameter squared.



3 Extraction of elastic constants from size effect data in bending of a circular cylindrical rod.
Rigidity/ diameter squared vs diameter squared.

Please see the original article for this figure.

4 Torsional deformation of negative Poisson's ratio foam. Line drawn across specimen discloses the warp. (a) Small cell size 0.3 mm; (b) larger cell size 0.8 mm.